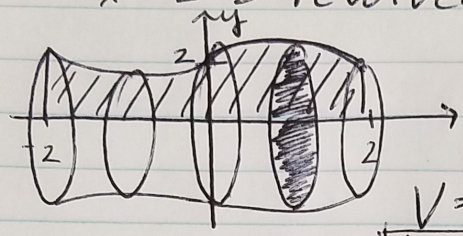
5.2

Solids revolved about an axis: Circular Cross-sections

EX #1 Find the volume of the solid w/ region bounded by $y = 2 + x\cos x$, $y = 0$, $x = -2$, $x = 2$ & revolved about the x-axis.



$A(x) = \pi r^2$

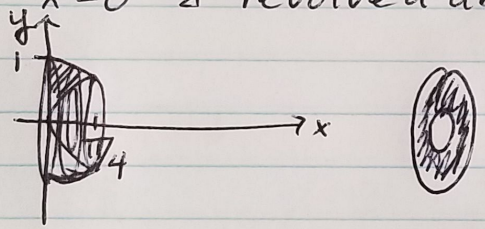
$A(x) = \pi (2 + x\cos x)^2$

$V = \pi \int_{-2}^{2} (2 + x\cos x)^2 \, dx$

$\boxed{V = 52.429 \text{ units}^3}$

Washers

EX #2 The region enclosed by $y = \sin x$, $y = \cos x$, $x = 0$ & revolved about x-axis. Find V.



$A(x) = \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$\neq \pi (R - r)^2$

$A(x) = \pi (\cos^2 x - \sin^2 x)$

$A(x) = \pi (\cos 2x)$

$V = \pi \int_0^{\pi/4} \cos 2x \, dx$

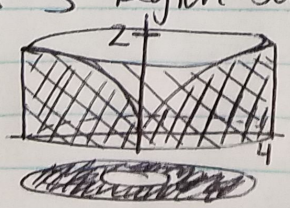
$= \frac{\pi}{2} \sin 2x \Big|_0^{\pi/4}$

$= \pi/2 (1 - 0)$

$\boxed{V = \pi/2 \text{ units}^3}$

Revolution about y-axis

EX #3 Region bounded by $y = \sqrt{x}$, $y = 0$, $x = 4$ & ↻ y-axis.



$y = \sqrt{x}$

$y^2 = x$

$A(y) = \pi (4^2 - y^4)$

$= \pi (16 - y^4)$

$V = \pi \int_0^2 (16 - y^4) \, dy$

$= \pi (16y - \frac{y^5}{5}) \Big|_0^2$

$= \pi (32 - \frac{32}{5})$

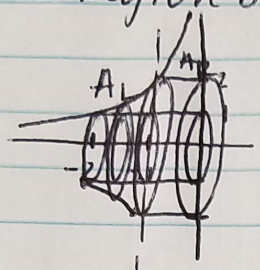
$= \pi (\frac{160}{5} - \frac{32}{5})$

$= \pi (\frac{128}{5})$

$\boxed{V = \frac{128}{5}\pi \text{ units}^3}$

Volume of Pieces

Ex #4 Region bounded by $y = -\frac{2}{x}$, $y = 1$, $y = 2$, $x = 0$, & x-axis.



$A(x)_L = \pi\left(\left(-\frac{2}{x}\right)^2 - 1^2\right)$ $A(x)_R = \pi(2^2 - 1^2)$

$$V = \pi\int_{-2}^{-1}\left(\frac{4}{x^2} - 1\right)dx + \pi\int_{-1}^{0} 3\,dx$$
$$= \pi\left(-\frac{4}{x} - x\right)\Big|_{-2}^{-1} + \pi(3x)\Big|_{-1}^{0}$$
$$= \pi(4 + 1 - (2 + 2)) + \pi(0 + 3)$$
$$= \pi + 3\pi$$

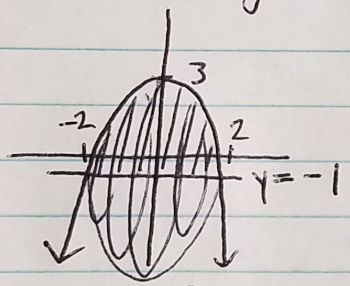
$2 = -\frac{2}{x}$ $1 = -\frac{2}{x}$

$x = -1$ $x = -2$

$$\boxed{V = 4\pi \text{ units}^3}$$

Not revolving about x- or y-axes.

Ex #5 Region bounded by $y = 3 - x^2$, $y = -1$, & $y = -1$.



$A(x) = \pi r^2$

$= \pi(3 - x^2 + 1)^2$

$= \pi(4 - x^2)^2$

$= \pi(16 - 8x^2 + x^4)$

$$V = \pi\int_{-2}^{2}(16 - 8x^2 + x^4)\,dx$$
$$= 2\pi\int_{0}^{2}(16 - 8x^2 + x^4)\,dx$$
$$= 2\pi\left(16x - \frac{8}{3}x^3 + \frac{x^5}{5}\right)\Big|_0^2$$
$$= 2\pi\left(32 - \frac{64}{3} + \frac{32}{5}\right)$$
$$= 2\pi\left(\frac{256}{15}\right)$$

$-1 = 3 - x^2$

$-4 = -x^2$

$4 = x^2$

$x = \pm 2$

$$\boxed{V = \frac{512}{15}\pi \text{ units}^3}$$