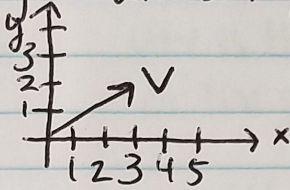
6.1 Vectors in the Plane

Vectors have magnitude & direction.

Notationally (in component form)

vector $v = \langle a, b \rangle$ is a vector starting @ the origin going a units right & b up.

$V = \langle 3, 2 \rangle$



zero vector has no length or direction & is represented by $v = \langle 0, 0 \rangle$

Head minus tail rule:

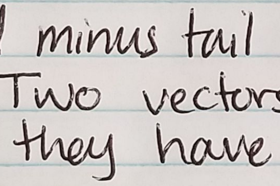
Two vectors are identical as long as they have the same length & direction.

Initial point (x_1, y_1) & terminal point (x_2, y_2) has vector $v = \langle x_2 - x_1, y_2 - y_1 \rangle$

Magnitude

$$|v| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Ex#1 Find the magnitude of the vector v represented by $\overrightarrow{PQ}$, with $P(-3, 4)$ & $Q(-5, 2)$.



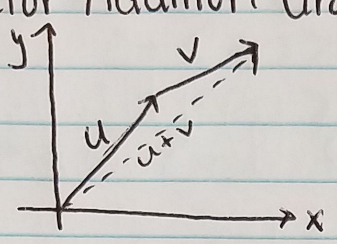
$$|v| = \sqrt{(-2)^2 + (-2)^2}$$
$$= \sqrt{8}$$
$$\boxed{|v| = 2\sqrt{2}}$$

Vector Operations $\quad v = \langle x_1, y_1 \rangle \quad u = \langle x_2, y_2 \rangle \quad K = \text{constant/scalar}$

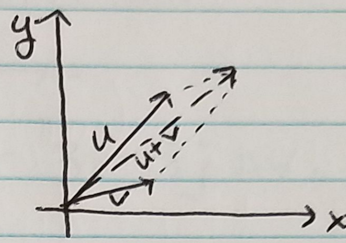
$v + u = \langle x_1 + x_2, y_1 + y_2 \rangle$

$Kv = K\langle x_1, y_1 \rangle = \langle Kx_1, Ky_1 \rangle$

Vector Addition Graphically



u+v is the hypotenuse of the triangle



u+v is the diagonal of the parallelogram

Ex#2 Let $u = \langle -1, 3 \rangle$ and $v = \langle 4, 7 \rangle$

a) $u + v = \langle 3, 10 \rangle$

b) $3u = \langle -3, 9 \rangle$

c) $2u + (-1)v = 2\langle -1, 3 \rangle + (-1)\langle 4, 7 \rangle$

$= \langle -2, 6 \rangle + \langle -4, -7 \rangle$

$= \langle -6, -1 \rangle$

Unit Vector

A vector with magnitude 1, generally represented by

$$u = \frac{v}{|v|} = \frac{1}{|v|} v$$

Ex#3 Find the unit vector in the direction $v = \langle -3, 2 \rangle$ and verify that it has a length/magnitude of 1.

$|v| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$

$u = \left(\frac{1}{\sqrt{13}}\right) \langle -3, 2 \rangle = \left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$

Check:

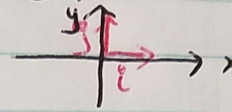
$|u| = \sqrt{\left(\frac{-3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2} = \sqrt{\frac{9}{13} + \frac{4}{13}} = \sqrt{\frac{13}{13}} = 1$

$u = \left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$

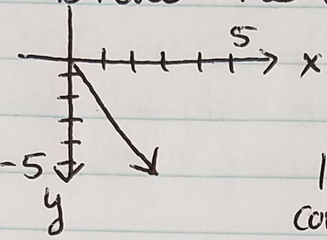
Vectors as a Linear Combination

$i = \langle 1, 0 \rangle$

$j = \langle 0, 1 \rangle$



Ex # 3.5 Draw the vector $3i - 5j = v$.



$v = 3i - 5j = \langle 3, -5 \rangle$

linear combination

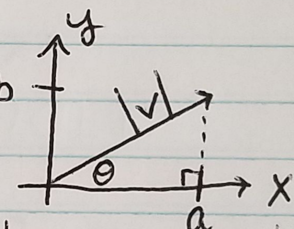
component form

Direction Angles

If $v = \langle a, b \rangle$ then

$a = |v| \cos\theta$

$b = |v| \sin\theta$



"resolve the vector" = determine values of a & b!

Ex #4 Find the linear components of the vector v with direction angle 115° and magnitude 6.

$a = 6 \cos 115° = -2.536$

$b = 6 \sin 115° = 5.438$

$v = -2.536i + 5.438j$

Ex #5 Find the magnitude and direction of

a) $u = \langle 3, 2 \rangle$

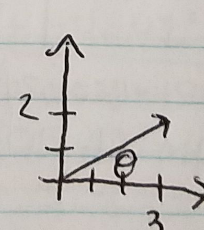
$|u| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$a = |u| \cos\theta$

$3 = \sqrt{13} \cos\theta$

$\theta = \cos^{-1}(3/\sqrt{13})$

$\theta \approx 33.690°$



$\boxed{|u| = \sqrt{13} \quad \theta \approx 33.690°}$

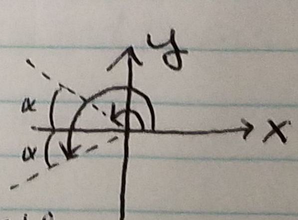
b) $v = \langle -2, -5 \rangle$

$|v| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$

$-2 = \sqrt{29} \cos\theta$

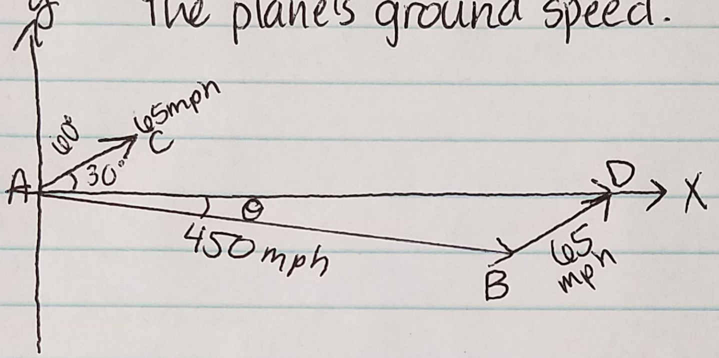
$\theta = \cos^{-1}(-2/\sqrt{29}) \approx 111.801°$

$\theta \approx 248.199$



Applications

Ex #6 A plane is about to take off to fly due east and there is a 65 mph wind with a bearing of 60 degrees. The plane's speed w/ no wind resistance is 450 mph. Find the compass heading (bearing) that the plane should go on & determine the plane's ground speed.



want $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$

$\overrightarrow{AB} = \langle 450\cos\theta, 450\sin\theta \rangle$

$\overrightarrow{AC} = \langle 65\cos 30, 65\sin 30 \rangle$

$\overrightarrow{AD} = \langle 450\cos\theta + 65\cos 30, 450\sin\theta + 65\sin 30 \rangle$

know $b = 0$ since flying <u>east</u>

$0 = 450\sin\theta + 65\sin 30$

$450\sin\theta = -65\sin 30$

$\theta = \sin^{-1}\left(\frac{-65\sin 30}{450}\right)$

$\theta \approx -4.142°$

compass heading $= 90° + |-4.142°| = 94.142°$

ground speed $= |\overrightarrow{AD}| = \sqrt{(65\cos 30 + 450\cos(-4.142))^2 + (450\sin(-4.142) + 65\sin 30)^2}$

$= 505.12$ mph

The plane must go on a heading of 94.142° & have a ground speed of 505.12 mph.