

7.9 Vectors

Vector: represents magnitude & direction ($v \in F$)

$\vec{v} = \langle a_1, a_2 \rangle = \overrightarrow{AB}$ where $A(x, y) \& B(x+a_1, y+a_2)$

magnitude: length of the vector

$$\|\vec{v}\| = \sqrt{a_1^2 + a_2^2}$$

zero vector: vector of magnitude zero or a vector w/ no magnitude & direction.

unit vector: vector of magnitude 1.

Additional notation: Standard basis vector notation

$$\langle 1, 0 \rangle = \vec{i}$$

$$\langle 0, 1 \rangle = \vec{j}$$

$$\vec{w} = \langle 2, 7 \rangle$$

$$\vec{w} = 2\vec{i} + 7\vec{j}$$

$$\vec{w} =$$

$$\langle a, b \rangle$$

$$\|\vec{w}\| = ?$$

$$\frac{\vec{w}}{\|\vec{w}\|} = \hat{w}$$

Properties

Addition: $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$

Subtraction: $\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$

Scalar multiplication: $c\vec{a} = \langle ca_1, ca_2 \rangle$

Parallel vectors: one vector will be a scalar multiple of the other.

Additional Props: $\vec{v} + \vec{w} = \vec{w} + \vec{v}$ $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

$$\vec{v} + \vec{0} = \vec{v}$$

$$1\vec{v} = \vec{v}$$

$$a(\vec{v} + \vec{w}) = a\vec{v} + a\vec{w}$$

$$(atb)\vec{v} = a\vec{v} + b\vec{v}$$

Dot Product: AKA scalar product or inner product

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

* nota vector *

More props: $\vec{u} \cdot (\vec{w} + \vec{v}) = \vec{u} \cdot \vec{w} + \vec{u} \cdot \vec{v}$ $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

$$(c\vec{v}) \cdot \vec{w} = \vec{v} \cdot (c\vec{w}) = c(\vec{v} \cdot \vec{w})$$

$$\vec{v} \cdot \vec{0} = 0$$

$$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$$

Angle b/t two vectors: $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos \theta$

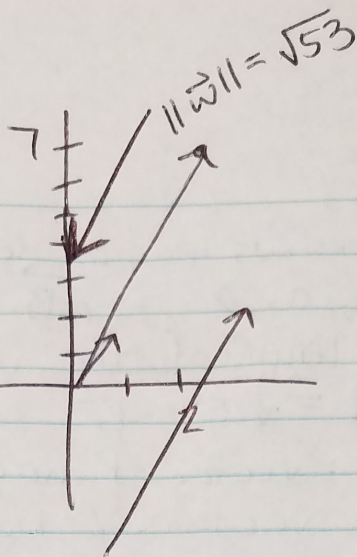
Orthogonal vectors: perpendicular vectors they will have a dot product = 0

Parallel vectors: $\theta = 0^\circ$ or $\theta = 180^\circ$, which means $\vec{a} \cdot \vec{b} = \pm \|\vec{a}\| \|\vec{b}\|$

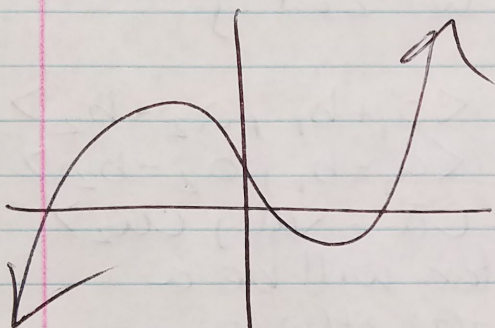
Vector-defined Functions: are a set of vectors whose endpts represent (x, y) , sort of like a parametric function.

$$\vec{w} = \langle 2, 7 \rangle$$

$$\frac{\vec{w}}{\|\vec{w}\|} = \left\langle \frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}} \right\rangle$$



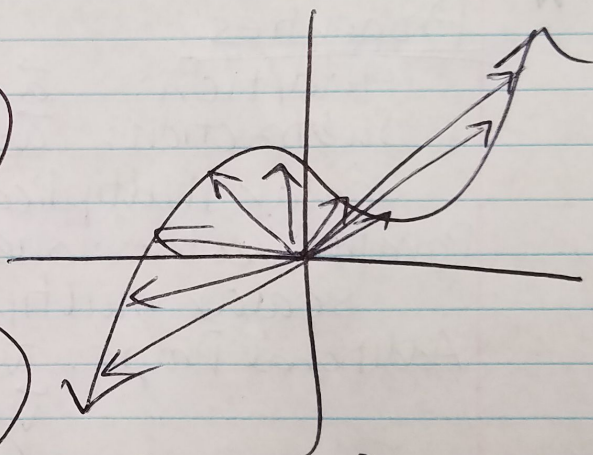
Parametrics



$$x = f(t)$$

$$y = g(t)$$

Vector Functions



$$F(x) = f(t)\vec{i} + g(t)\vec{j}$$