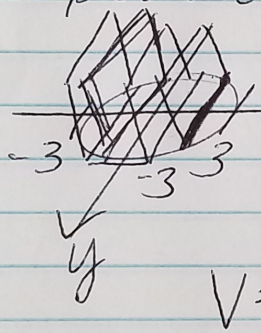
5.1

The volume of a solid of known integrable cross-sections of area $A(x)$ from $[a,b]$ is

$$V = \int_a^b A(x)\,dx$$

EX #1

Find the volume of the solid whose base is bounded by the circle $x^2+y^2=9$ and the cross sections perpendicular to the x-axis are squares.



$A(x) = \text{base}^2$

$A(x) = (2\sqrt{9-x^2})^2$

$$V = \int_{-3}^{3} (2\sqrt{9-x^2})^2\,dx$$

$$= 4\int_{-3}^{3} (9-x^2)\,dx$$

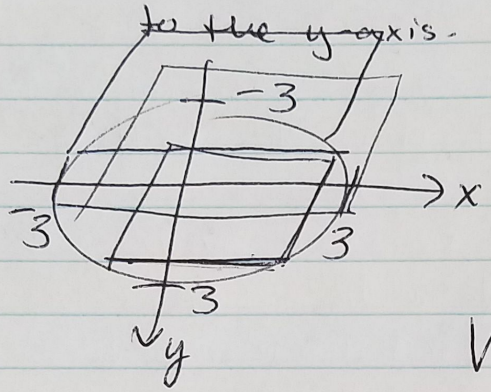
$$= 4\left(9x - \frac{x^3}{3}\right)\Big|_{-3}^{3}$$

$$= 4(27-9-(-27+9))$$

$$= 4(54-18)$$

$$\boxed{V = 144 \text{ units}^3}$$

Ex #2 Base is the circle $x^2+y^2=9$ & cross-sections are squares perpendicular ~~to the y-axis~~.



$A(y) = \text{base}^2$

$A(y) = (2\sqrt{9-y^2})^2$

$x^2+y^2=9$

$x=\sqrt{9-y^2}$

$$V = \int_{-3}^{3} A(y)\,dy$$

$$V = 4\int_{-3}^{3}(9-y^2)\,dy$$

$$V = 8\int_{0}^{3}(9-y^2)\,dy$$

$$\vdots$$

$$\boxed{V = 144 \text{ units}^3}$$