4.3/4.4

u-Substitution (Int. tech #1)

Ex#1 $\int \frac{2x^2}{(4+x^3)^2}\,dx = 2\int \frac{1}{3}\left(\frac{1}{u^2}\right)du$ $\quad u = 4+x^3$ $\quad du = 3x^2\,dx$, $\frac{1}{3}du = x^2\,dx$

$= \frac{2}{3}\int \frac{1}{u^2}\,du$

$= \frac{2}{3}\left(-\frac{1}{u}\right) + C$

$= \boxed{\frac{2}{3(4+x^3)} + C}$

Ex#2 $\int \sin\frac{x}{2}\,dx = 2\int \sin u\,du$ $\quad u = \frac{x}{2}$ $\quad du = \frac{1}{2}dx$, $2du = dx$

$= -2\cos u + C$

$= \boxed{-2\cos\frac{x}{2} + C}$

Ex#3 $\int x^4\sqrt{3x^5-4}\,dx = \frac{1}{15}\int \sqrt{u}\,du$ $\quad u = 3x^5-4$ $\quad du = 15x^4\,dx$, $\frac{1}{15}du = x^4\,dx$

$= \frac{1}{15}\left(\frac{2}{3}\right)u^{3/2} + C$

$= \boxed{\frac{2}{45}(3x^5-4)^{3/2} + C}$

Ex#4 $\int x\sqrt{x+1}\,dx$

$u = \sqrt{x+1}$

$u^2 = x+1$

$u^2 - 1 = x$ $\quad 2u\,du = dx$

$= \int (u^2-1)u(2u)\,du$

$= \int (2u^4 - 2u^2)\,du$

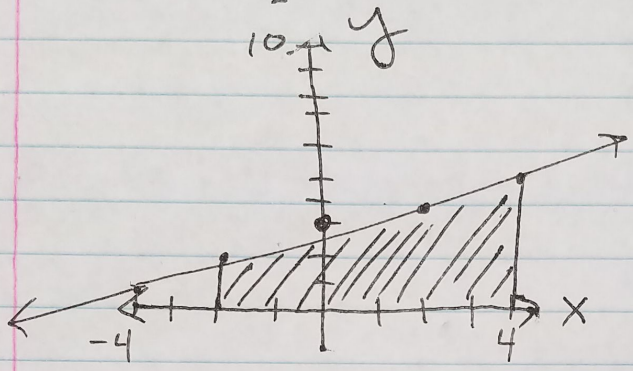
$= \frac{2}{5}u^5 - \frac{2}{3}u^3 + C = \boxed{\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C}$

Definite Integrals

Evaluating the integran on $[a, b]$.

Ex #5 $\int_{-2}^{4} \left(\frac{x}{2}+3\right)dx = \frac{b_1+b_2}{2}h = \frac{2+5}{2}(6)$

$= \boxed{21}$



$$\int_{-2}^{4} \left(\frac{x}{2}+3\right)dx = \left[\frac{1}{4}x^2+3x+C\right]_{-2}^{4}$$

$$= \left(\frac{16}{4}+12+\cancel{C}\right) - \left(\frac{4}{4}-6+\cancel{C}\right)$$

$$= \boxed{21}$$

Some Integration Rules

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b \left[f(x)+g(x)\right]dx = \int_a^b f(x)dx + \int_a^b g(x)\,dx$$

If $a < b < c$,

$$\int_b^c f(x)dx = \int_a^c f(x)dx - \int_a^b f(x)dx$$