Simple Harmonic Motion

An object that vibrates/oscillates back & forth around a point (point of equilibrium) is in simple harmonic motion. Examples of this are: a weight attached to a spring, a pendulum, a string on an instrument, etc.

$y = A\sin \omega t$ $\quad$ $y = A\cos \omega t$

y = position of the object relative to equilibrium

$|A|$ = maximum distance from equilibrium (amplitude)

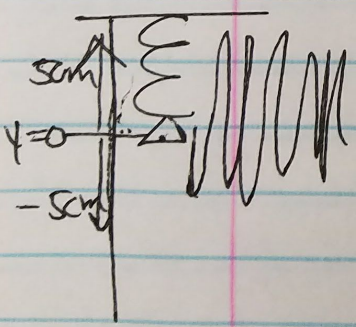
ω = angular frequency / angular velocity

$\omega = 2\pi f$

f = # of oscillations per unit of time

$f = \frac{\omega}{2\pi}$

Ex#3 A mass oscillating up & down on the bottom of a spring can be modeled as harmonic motion. If the weight is displaced a maximum of 5 cm from equilibrium, find the modeling equation if it takes 2 seconds to complete one cycle. Assume it starts @ equilibrium.



Using sine b/c mass is @ equilibrium & defining down as negative & up as positive.

$y = -5\sin \pi t$

$|A| = 5$ cm

$\omega = \pi$ rad/s

$f = \frac{\omega}{2\pi}$

$f = \frac{1 \text{ cycle}}{2 \text{ sec}}$

$\omega = 2\pi f$ or $\frac{1}{2} = \frac{\omega}{2\pi}$ $\rightarrow$ $\omega = \pi$

Ex #4 A white dot is painted on the outer rim of a bicycle tire. The dot rotates at a rate of 80 rpm & oscillates between 1 & 19 inches off of the ground. Write an equation that represents the height h in inches of the dot at any second t if the dot starts @ the bottom.

$|A| = \frac{19-1}{2} = \frac{18}{2} = 9$ inches

$\omega = 2\pi f = 2\pi\left(\frac{80 \text{ rev}}{1 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 2\pi\left(\frac{8}{6}\right) = 2\pi\left(\frac{4}{3}\right) = \frac{8\pi}{3}$ rad/sec

$y = 10$ inches is the mid line

$h(t) = -9\cos\left(\frac{8\pi}{3}t\right) + 10$

How high is the dot after 3 sec?

$h(3) = -9\cos\left(\frac{8\pi}{3}\cdot 3\right) + 10$

$= -9\cos(8\pi) + 10$

$= -9(1) + 10$

$h(3\text{sec}) = 1$ inch