4.2 Trigonometric Functions

Trig Ratios

$\sin\theta = o/h$ $\quad\quad$ $\csc\theta = h/o$

$\cos\theta = a/h$ $\quad\quad$ $\sec\theta = h/a$

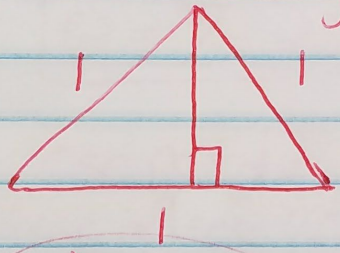
$\tan\theta = o/a$ $\quad\quad$ $\cot\theta = a/o$

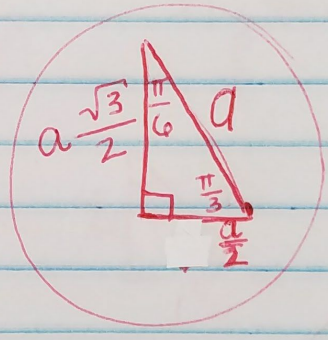
Special Angles

$45° = \frac{\pi}{4}$ rad $\quad\quad$ $60° = \frac{\pi}{3}$ rad $\quad\quad$ $180° = \pi$ rad

$30° = \frac{\pi}{6}$ rad $\quad\quad$ $90° = \frac{\pi}{2}$ rad $\quad\quad$ $360° = 2\pi$ rad

Special Triangles



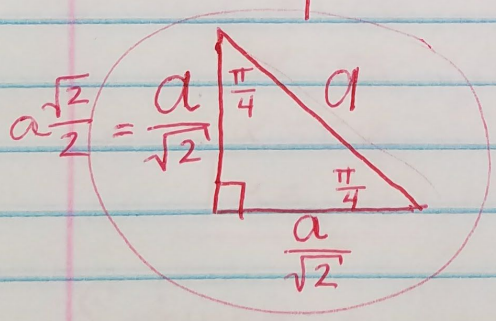


$a^2 + \left(\frac{1}{2}\right)^2 = 1^2$

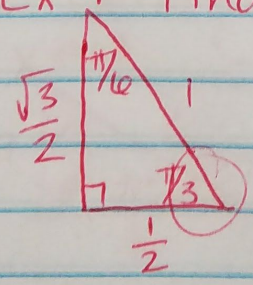
$a^2 = 1 - \frac{1}{4}$

$a^2 = \frac{3}{4}$

$a = \frac{\sqrt{3}}{2}$



Ex#1 Find all 6 trig ratios for $\theta = 60° = \frac{\pi}{3}$



$\sin(\pi/3) = o/h = \frac{\sqrt{3}/2}{1} = \frac{\sqrt{3}}{2}$

$\cos(\pi/3) = a/h = \frac{1/2}{1} = 1/2$

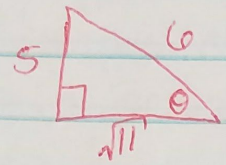
$\tan(\pi/3) = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2}\left(\frac{2}{1}\right) = \sqrt{3}$

$\csc(\pi/3) = h/o = \frac{1}{\sqrt{3}/2} = 1\left(\frac{2}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

$\sec(\pi/3) = 2$

$\cot(\pi/3) = \frac{1}{\sqrt{3}}$

Ex #2 Find all 6 ratios if $\sin\theta = 5/6$.



$\sin\theta = 5/6$ $\quad \csc\theta = 6/5$

$\cos\theta = \sqrt{11}/6$ $\quad \sec\theta = 6/\sqrt{11}$

$\tan\theta = 5/\sqrt{11}$ $\quad \cot\theta = \sqrt{11}/5$

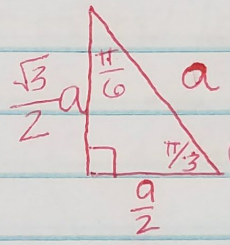
Ex #3 What angles make $\cos\theta = 1/2$ true? What about $\csc\alpha = \sqrt{2}$?

$\cos\theta = 1/2$

$\cos^{-1}[\cos\theta] = \cos^{-1}(1/2)$

$\theta = \cos^{-1}(1/2)$

$\theta = \pi/3$ or $\theta = 60°$



$\cos\frac{\pi}{3} = \frac{adj}{hyp} = \frac{a/2}{a}$

$\cos\frac{\pi}{3} = 1/2$ ✓

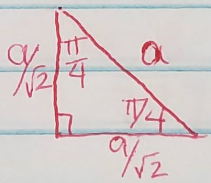
$\csc\alpha = \sqrt{2}$

$\csc^{-1}[\csc\alpha] = \csc^{-1}(\sqrt{2})$

$\alpha = \csc^{-1}(\sqrt{2})$

(or $\alpha = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$)

$\alpha = \pi/4$ or $\alpha = 45°$



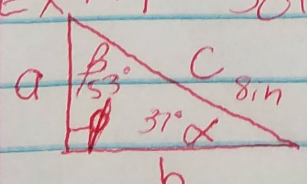
$\csc\left(\frac{\pi}{4}\right) = \frac{hyp}{opp} = \frac{a}{a/\sqrt{2}} = a\left(\frac{\sqrt{2}}{a}\right)$

$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$ ✓

Solving a Triangle

Determine the values for all angles & sides.

Ex #4 Solve Δ if $\alpha = 37°$ & $c = 8$ in.



$\alpha + \beta = 90°$

$37° + \beta = 90°$

$\beta = 53°$

$\sin\alpha = \frac{a}{c}$

$\sin 37° = \frac{a}{8in.}$

$a = 8\sin 37°$

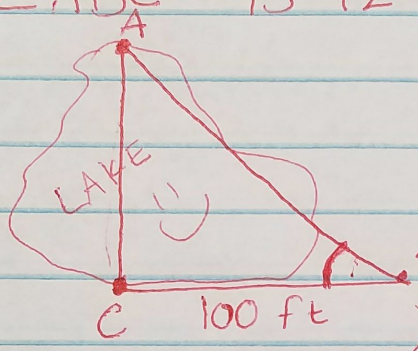
$a = 4.815$ in

$\cos 37° = \frac{b}{8}$

$b = 6.389$ in

$\alpha = 37°$, $\beta = 53°$, $a = 4.815$ in, $b = 6.389$ in, $c = 8$ in

Ex #5 (#65 on HW) DeShanda's surveyor team wants to determine the length AC. One person positioned at A, other at C and DeShanda at B w/ angle-measuring instrument (100 ft from C). She measured $\angle ABC = 75°12'42''$.



$$\angle ABC = 75°12'42''$$
$$= 75° + 12'\left(\frac{1°}{60'}\right) + 42''\left(\frac{1'}{60''}\right)\left(\frac{1°}{60'}\right)$$
$$= 75° + .2° + .0116\overline{66}$$
$$\angle ABC = 75.211\overline{666}°$$

$$\tan(\angle ABC) = \frac{\overline{AC}}{100\text{ ft}}$$
$$100 \cdot \tan(75.211\overline{666}) = \overline{AC}$$
$$\boxed{\overline{AC} = 378.797\text{ ft}}$$