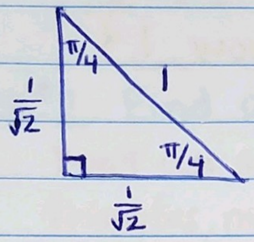
4.2 Trigonometric Functions

Special Angles

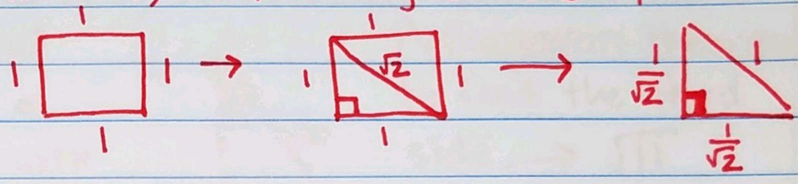
$30° = \pi/6$ $60° = \pi/3$ $45° = \pi/4$ $90° = \pi/2$

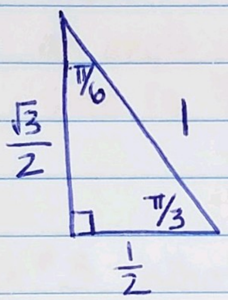
(and obviously $180° = \pi$ is also special)

Special Triangles

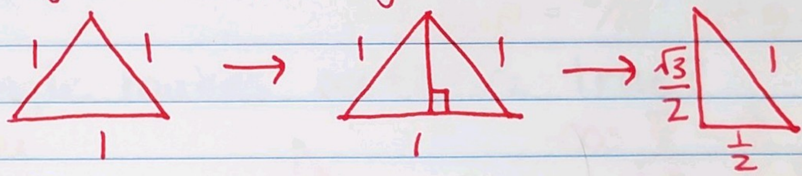


This comes from taking a square with side lengths 1, determening length of diagonal with pythagorean theorem, then dividing all sides by $\sqrt{2}$ on the triangle.





This comes from taking an equilateral triangle with side lengths 1, spitting it in half, then finding the height with pythagorean theorem.



Trigonometric Ratios

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

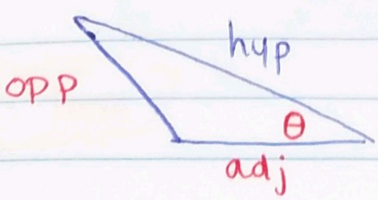
an easy way to remember this is the word "SOH-CAH-TOA"

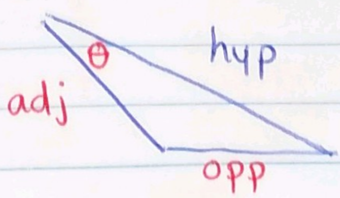
$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Sine and cosecant are reciprocals (sin & csc).
Cosine and secant are reciprocals (cos & sec).
Tangent and cotangent are reciprocals (tan & cot).

What is "opposite/adjacent/hypotenuse"

Opposite and adjacent depend completely on the acute angle you are working with. Hypotenuse is always the longest side, opposite to the ~~right~~ largest angle.



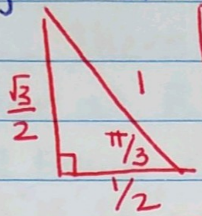


We're only working with right triangles today

You should be able to do #1–4 without calculators!

Ex #1 Find all 6 trigonometric ratios for $\theta = \pi/3$.

Use the special triangle with $\pi/3$ in it.



$\sin\frac{\pi}{3} = \sqrt{3}/2$ $\csc\frac{\pi}{3} = 2/\sqrt{3}$

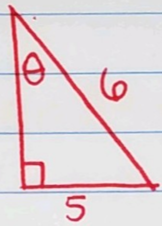
$\cos\frac{\pi}{3} = 1/2$ $\sec\frac{\pi}{3} = 2$

$\tan\frac{\pi}{3} = \sqrt{3}$ $\cot\frac{\pi}{3} = 1/\sqrt{3}$

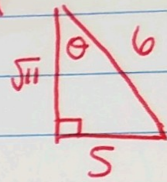
{I won't require you to rationalize denominators.}

Ex #2 Find all 6 trig ratios if $\sin\theta = 5/6$.

Don't use special triangles, make a right triangle with an angle whose sine is 5/6 (opp/hyp).



Pythagorean theorem to find the third side → $\sqrt{11}$



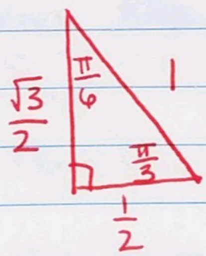
$\sin\theta = 5/6$ $\csc\theta = 6/5$

$\cos\theta = \sqrt{11}/6$ $\sec\theta = 6/\sqrt{11}$

$\tan\theta = 5/\sqrt{11}$ $\cot\theta = \sqrt{11}/5$

Ex #3 What angle makes $\cos\theta = 1/2$ true?

Only using the special triangles, guess and check the anlge that will make this true.



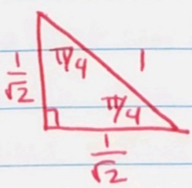
$\cos\frac{\pi}{3} = \frac{1}{2}$

so $\theta = \frac{\pi}{3}$ is the only one to work

Ex #4 What angle makes $\csc\alpha = \sqrt{2}$ true?

↑ another greek letter, "alpha"

Guess and check with special triangles again!



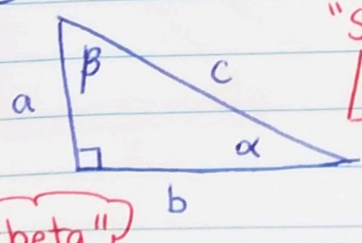
$\csc\frac{\pi}{4} = \sqrt{2}$

$\alpha = \pi/4$

{for now/today only one angle will work!}

Ex #5 Solve the triangle if $\alpha = 37°$ and $c = 8$ inches.

"Solve the triangle" means find all sides & angles*



{β = "beta"}

$\beta = 53°$ since angles need to add to 180°

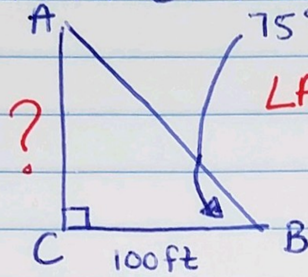
$\sin\alpha = \frac{a}{c}$ $\cos\alpha = \frac{b}{c}$

$\sin 37 = \frac{a}{8}$ use a calculator on degree mode $\cos 37 = \frac{b}{8}$

$a = 8\sin 37$ $b = 8\cos 37$

$a = 4.815$ inches $b = 6.389$ inches

Ex #6 Danny's surveyor team wants to find the length of $\overline{AC}$. There is one person positioned at A, another at B, and person B is 100 ft from C. We know $\angle ABC = 75°12'42''$.



$$75°12'42'' = 75° + 12'\left(\frac{1°}{60'}\right) + 42''\left(\frac{1°}{3600''}\right)$$

$$\angle ABC = 75.211666°$$

(use more decimals than needed in the middle of the problem)

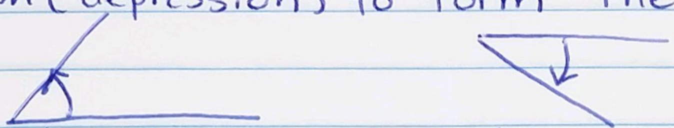
$$\tan B = \frac{?}{100}$$

$$\overline{AC} = ? = 100 \tan 75.211666$$

$$\boxed{\overline{AC} = ? = 378.797 \text{ ft}}$$

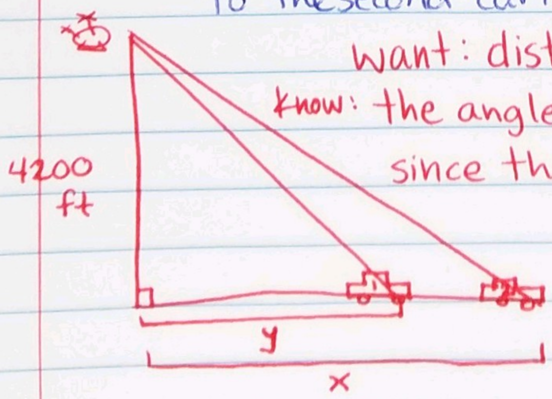
Angles of Elevation & Depression

From a horizontal line go up (elevation) or down (depression) to form the angle.



elevation depression

Ex #7 A police helecopter is monitoring the speed of two cars on a straight road. The police are at an altitude of 4200 ft straight above the road. At one instant, the angle of elevation from the first car to the police is 20°, and to the second car its 15°. How far apart are the two cars?



want: distance between cars = x − y

know: the angle at car 1 is 20° & at car 2 it 15° since the smaller angle must be farther away.

$$\tan 20° = \frac{4200}{y} \qquad \tan 15° = \frac{4200}{x}$$

$$y = \frac{4200}{\tan 20°} \qquad x = \frac{4200}{\tan 15°}$$

$$x - y = \frac{4200}{\tan 15°} - \frac{4200}{\tan 20°}$$

$$\boxed{\text{distance b/t cars} = x - y = 4135.208 \text{ ft}}$$