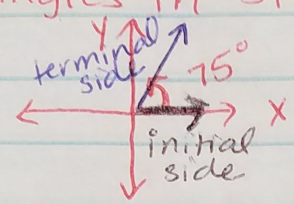
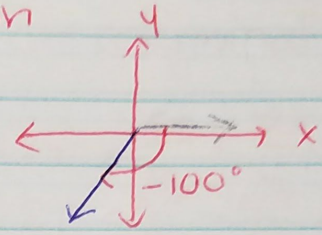
(Unit Circle)

4.3 Trigonometry Extended: The Circular Functions

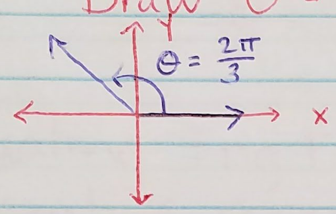
Angles in Standard Position





positive angle   negative angle

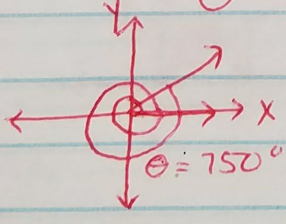
Ex #1 Draw $\theta = \frac{2\pi}{3}$ radians in standard position.



Coterminal Angles

Angles that have the same inital & terminal side, but different angles of rotation.

Ex #2 Find 7 angles that are coterminal with $\theta = 750°$.



angles coterm. w/ $\theta = 750°$

$\theta = 30°$

$\theta = 390°$

$\theta = -330°$

$\theta = -690°$

$\theta = 1110°$

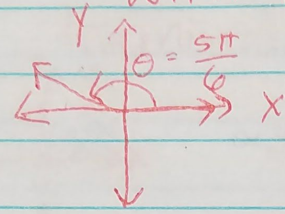
$\theta = 1470°$

$\theta = 1830°$

neg: $750° - 360° - 360° - 360° = -330°$

pos: $750° + 360° =$ ~~[illegible]~~ $1110°$

Ex #3 Find 5 angles that are coterminal with $\theta = \frac{5\pi}{6}$.



pos: $\frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

neg: $\frac{5\pi}{6} - 2\pi = \frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

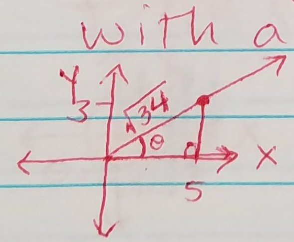
5 coterminal angles

$\theta = \frac{17\pi}{6}$, $\theta = \frac{-7\pi}{6}$, $\theta = \frac{29\pi}{6}$, $\theta = \frac{-19\pi}{6}$, $\theta = \frac{41\pi}{6}$

Evaluating Trig Functions on the Coordinate Plane

Negative coordinates correspond to negative "lengths" on triangles.
Hypotnuse is always positive.

Ex #4 Find all 6 trig ratios for the acute angle in standard position with a terminal side going through (5,3).



$a^2 + b^2 = c^2$

$5^2 + 3^2 = c^2$

$25 + 9 = c^2$

$34 = c^2$

$\sqrt{34} = c$

$\sin\theta = \frac{3}{\sqrt{34}}$

$\cos\theta = 5/\sqrt{34}$

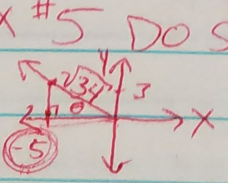
$\tan\theta = 3/5$

$\cot\theta = 5/3$

$\sec\theta = \sqrt{34}/5$

$\csc\theta = \sqrt{34}/3$

Ex #5 Do same for (-5, 3).

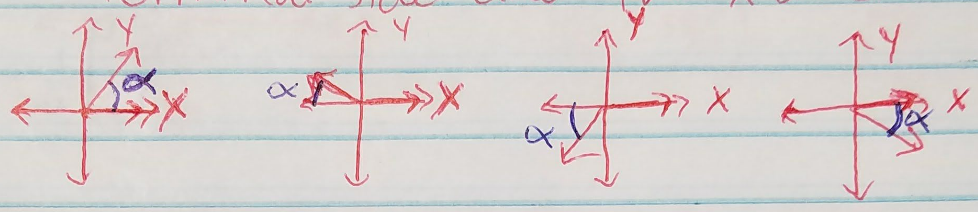


$\sin\theta = 3/\sqrt{34}$ $\quad \tan\theta = -3/5$ $\quad \sec\theta = \frac{-\sqrt{34}}{5}$

$\cos\theta = -5/\sqrt{34}$ $\quad \cot\theta = -5/3$ $\quad \csc\theta = \frac{\sqrt{34}}{3}$

Reference Angles

The acute angle between the terminal side and the x-axis.



Ex #6 Use a reference angle and special right triangles to determine the following ratios.

a) $\sin(-315°)$

b) $\cos(5\pi/3)$

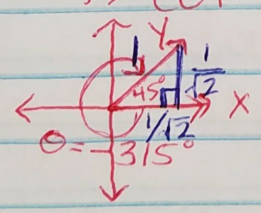
c) $\tan(210°)$

d) $\csc(-3\pi/4)$

e) $\sec(330°)$

~~f) $\cot(15\pi/6)$~~

a)

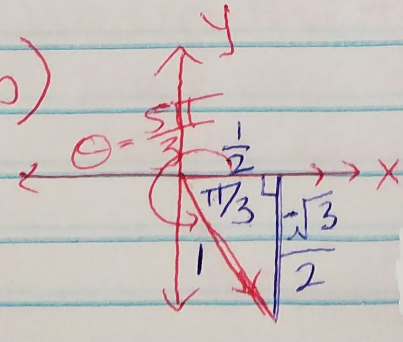


$\sin(-315°) = \cancel{\sin(45°)}$

$= 1/\sqrt{2}$

$\boxed{\sin(-315°) = \frac{1}{\sqrt{2}}}$

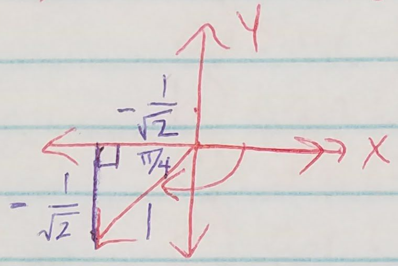
b)



$\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

$\boxed{\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}}$

d) ~~sec~~ csc$(-3\pi/4)$



$\csc(-3\pi/4) = \frac{hyp}{opp}$

$= \frac{1}{-1/\sqrt{2}}$

$\boxed{\csc(-3\pi/4) = -\sqrt{2}}$

Using One Ratio to Find the Others.

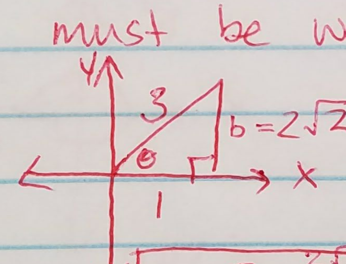
Ex #7 Find all 6 trig ratios if

$\sec\theta = 3$ & $\sin\theta > 0$.

since $\sin\theta > 0$ it means it is in QI or QII.

since $\sec\theta = 3$ (>0) it means it is in QI or QIV.

must be working in QI



$\sec\theta = 3/1 = hyp/adj$

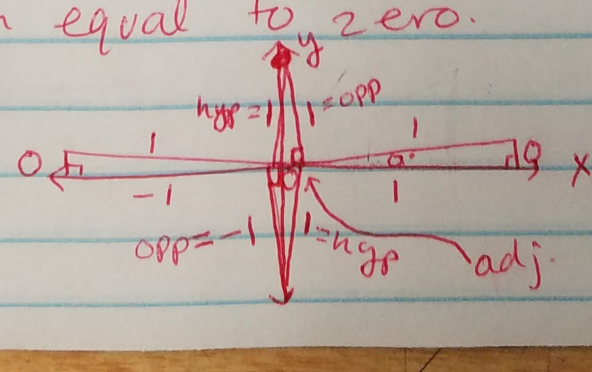
$1^2 + b^2 = 3^2$

$b = \sqrt{8} = 2\sqrt{2}$

$\sin\theta = \frac{2\sqrt{2}}{3}$	$\csc\theta = \frac{3}{2\sqrt{2}}$
$\cos\theta = \frac{1}{3}$	$\sec\theta = 3$
$\tan\theta = 2\sqrt{2}$	$\cot\theta = \frac{1}{2\sqrt{2}}$

Quadrential Angles

must use "triangles" with one side length equal to zero.



Ex #8 Determine

a) $\sin(-270^\circ)$

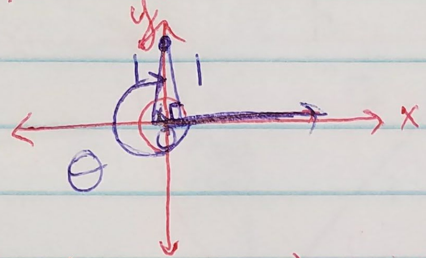
b) $\tan(3\pi)$

c) $\sec(11\pi/2)$

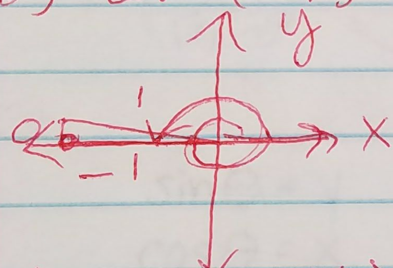
d) $\cot(15\pi/6)$

a) $\sin(-270^\circ)$

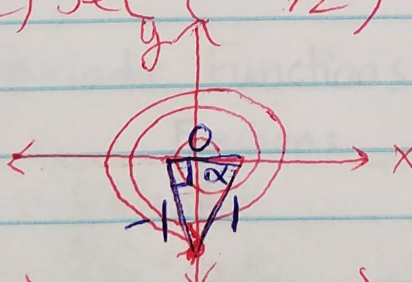
$\sin(-270^\circ) = \frac{opp}{hyp} = \frac{1}{1} = 1$



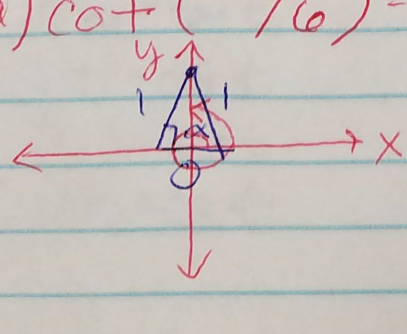
b) $\tan(3\pi) = \frac{opp}{adj} = \frac{0}{-1} = 0$



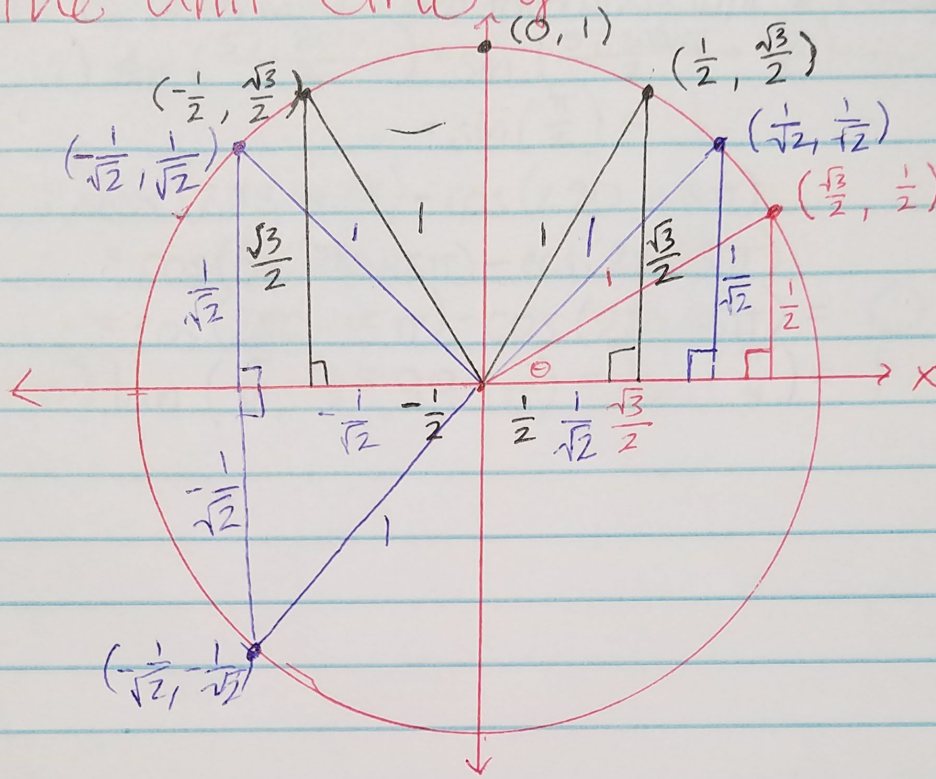
c) $\sec(11\pi/2) = \frac{hyp}{adj} = \frac{1}{0} =$ undefined



d) $\cot(15\pi/6) = \cot(5\pi/2) = \frac{adj}{opp} = \frac{0}{1} = 0$



The Unit Cirle



$\sin\theta = y \qquad \tan\theta = \frac{y}{x} \qquad \sec\theta = \frac{1}{x}$

$\cos\theta = x \qquad \cot\theta = \frac{x}{y} \qquad \csc\theta = \frac{1}{y}$

Periodic Functions (AKA Circular Functions)

Functions	Period
$f(t) = \sin t$	$T = 360° = 2\pi$
$f(t) = \cos t$	$T = 360° = 2\pi$
$f(t) = \tan t$	$T = 180° = \pi$ rad
$f(t) = \cot t$	$T = 180° = \pi$ rad
$f(t) = \sec t$	$T = 360° = 2\pi$
$f(t) = \csc t$	$T = 360° = 2\pi$

Ex #9 Determine the following w/o a calculator

a) $\sin\left(\frac{57,801\pi}{2}\right) = \sin\left(\frac{57,800\pi}{2} + \frac{1\pi}{2}\right) = \sin\left(28,900\pi + \frac{\pi}{2}\right)$

$= \sin\left(\frac{\pi}{2}\right) = 1$

b) $\cos(288.45\pi) - \cos(280.45\pi)$

$= \cos(8\pi + 280.45\pi) - \cos(280.45\pi)$

$= \cos(280.45\pi) - \cos(280.45\pi) = 0$

c) $\tan\left(\frac{\pi}{4} - 99,999\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1$