4.6: Graphs of Composite Trigonometric Functions

Objective: Perform combinations and compositions of trigonometric functions with other families of functions (rational, exponential, polynomial, etc.)

Ex #1: Graph the following functions over $[-2\pi, 2\pi]$, adjusting the vertical window as needed. Which appear to be periodic?

a) $y = \sin x + x^2$ NOPE

b) $y = x^2 \sin x$ NOPE

c) $y = (\sin x)^2$ Yes! Periodic

d) $y = \sin(x^2)$ NOPE

Proving Periodicity: use the fact that $\sin(x+2\pi) = \sin x$

Ex #2: Prove algebraically that $f(x) = (\sin x)^2$ is periodic & determine its period graphically.

$$f(x+2\pi) = f(x)$$
$$(\sin(x+2\pi))^2 =$$
$$(\sin x)^2 =$$
$$f(x) = f(x) \quad \blacksquare$$

f(x) must be periodic w/ some period that divides 2π

$\boxed{T_{f(x)} = \pi}$

Ex #3: Prove algebraically that $f(x) = \sin^3 x$ is periodic and determine its period graphically. State the domain & range & sketch 2 periods.

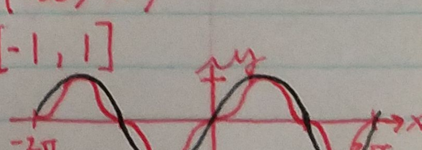
Compare $\sin x$ to $\sin^3 x$. $\sin^3 x = (\sin x)^3$

$$f(x+2\pi) = f(x)$$
$$\sin^3(x+2\pi) =$$
$$[\sin(x+2\pi)]^3 =$$
$$[\sin x]^3 =$$
$$\sin^3 x =$$
$$f(x) = f(x) \quad \blacksquare$$

$T = 2\pi$

D: $(-\infty, \infty)$

R: $[-1, 1]$



$\sin x$ & $\sin^3 x$ have the same zeros, period, amp, extrema

$\sin^3 x$ looks like a cubic @ each zero

$\sin^3 x$ is closer to the x-axis because $|y^3| \le |y|$ if $|y| \le 1$

Nonnegative Periodic Functions (absolute value)

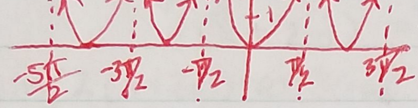
Ex #4: Determine the domain, range, & period of the following. sketch a graph w/ 4 periods.

a) $f(x) = |\tan x|$

D: $\{x \mid x \neq \frac{\pi}{2} + \pi n, n \in \mathbb{Z}, x \in \mathbb{R}\}$

R: $[0, \infty)$

$T = \pi$

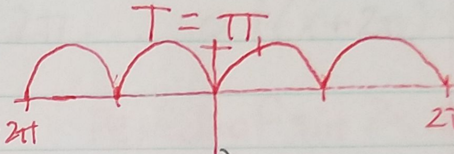


b) $f(x) = |\sin x|$

D: $(-\infty, \infty)$

R: $[0, 1]$

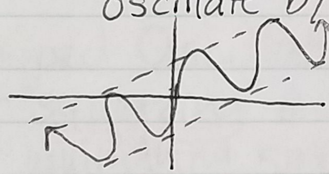
$T = \pi$



Sinusoid and Linear (nonconstant) Combination

– oscillation b/t parallel lines –

Ex #5: What two parallel lines does $f(x) = 0.5x + \sin x$ oscillate b/t?



Since $\sin x$ (alone) oscillates b/t $[-1, 1]$, $f(x)$ will osaillate b/t $0.5x - 1$ & $0.5x + 1$.

Parallel lines: $y = 0.5x \pm 1$

Sums & Differences of Sinusoids

- sums & differences of sinusoids w/ the same period are again sinusoids –

Ex #6: Identify which of the following are sinusoids

a) $5\cos x + 3\sin x$ Yes, $T = 2\pi$

b) $\cos 5x + \sin 3x$ Nope

c) $2\cos 3x - 3\cos 2x$ Nope

d) $a\cos(\frac{3x}{7}) - b\cos(\frac{3x}{7}) + c\sin(\frac{3x}{7})$ Yes, $T = 14\pi/3$

Ex #7: $f(x) = 2\sin x + 5\cos x$

a) Is it a sinusoid? Yes

b) What is the period? $T = 2\pi$

c) Estimate the amplitude & phase shift graphically (to the nearest hundredth)

5.38516480 → $a \approx 5.39$ $\quad \phi \approx -1.19 \leftarrow -1.19029$

d) Give a sinusoid in the form $a\sin(b(x-h)) + k$ that approximates $f(x)$. $f(x) = 5.39\sin(x + 1.19)$

Non Sinusoidal Periodic Functions

- keep in mind if $f(x)$ is periodic that
  $f(x+S)=f(x)$ where S is the period
  or some multiple of the period

Ex #8: Show $f(x)=\sin 2x+\cos 3x$ is periodic, but not sinusoidal. Graph one period.

guess that period is 2π . . . so $f(x+2\pi)=f(x)$

$f(x+2\pi) = f(x)$

$\sin(2(x+2\pi))+\cos(3(x+2\pi))=$

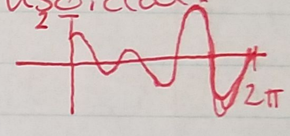
$\sin(2x+4\pi)+\cos(3x+6\pi)=$

$\sin(2x)+\cos(3x)=$

$f(x)=f(x)$ Periodic!

$T=2\pi$

Periods of $\sin 2x$ & $\cos 3x$ are not the same ∴ not sinusoidal.



Damped Oscillation

- "squeezing" @ origin or infinities when multiplying sine or cosine by another function (damping factor)

Ex #9: Identify which of the following exhibit damped oscillation, ID the damping factor, & ID where the damping occurs.

a) $f(x)=2^{-x}\sin 4x$ yes, $y=2^{-x}$, $x\to\infty$

b) $f(x)=3\cos 2x$ no

c) $f(x)=-2x\cos 2x$ yes, $y=-2x$, $x\to 0$

Ex #10 A physics class collected data for an air table glider that oscillates b/t two springs. The class determined from the data that the equation $y=0.22e^{-0.065t}\cos 2.4t$ modeled the displacement y of the spring from its original position as a function of time t.

a) ID damping factor & where it occurs

$y=0.22e^{-0.065t}$, $x\to\infty$

b) Approximate how long it takes for the spring to be damped so that y is b/t ± 0.1 @ $t\approx 11.859$ seconds