1.6 Transformations

$f(x) = x$, $f(x) = x^3$, $f(x) = x^2$, $f(x) = \frac{1}{x}$

$f(x) = \sqrt{x}$, $f(x) = \ln x$, $f(x) = e^x$, $f(x) = \sin x$,

$f(x) = \frac{1}{1+e^{-x}}$, $f(x) = \cos x$

Translations

$y = f(x-c)$ "a horizontal translation to the right by c units"

$y = f(x+c)$ "a horizontal translation to the left by c units"

$y = f(x) + c$ "a vertical translation up by c units"

$y = f(x) - c$ "a vertical translation down by c units"

Reflections

$y = -f(x)$ "a reflection across the x-axis"

$y = f(-x)$ "a reflection across the y-axis"

$y = -f(-x)$ "a reflection through the origin"

Ex #1 Describe how the graphs of f are transformed into the graphs of g.

$30 - x = 0$

$-x = -30$

$x = 30$ $\rightarrow$

a) $f(x) = x^3$ $g(x) = (30-x)^3$

The graph of f must be reflected across the y-axis and translated horizontally 30 units to the right.

b) $f(x) = x^2 - 1$ $g(x) = x^2 + 3$

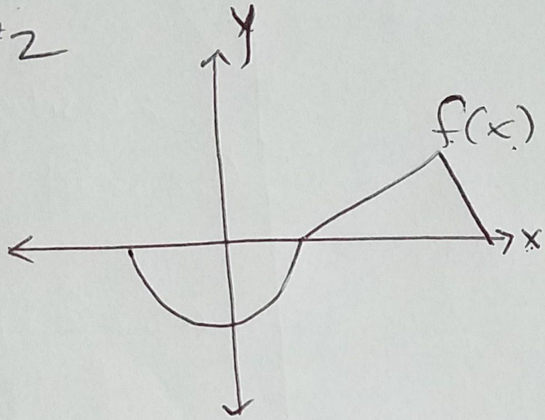
The graph of f must be translated vertically 4 units up.

Absolute Value Compositions

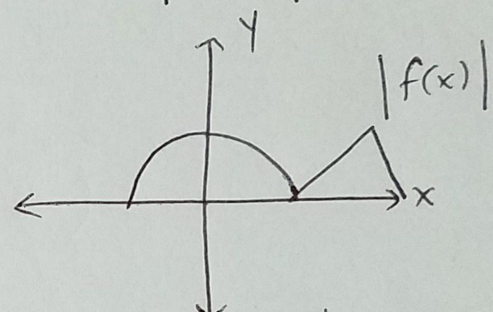
$y = |f(x)|$ Reflection of everything <u>under</u> the x-axis.

$y = f(|x|)$ Replaces what is on the <u>left</u> of the y-axis with a reflection of what is on the right.

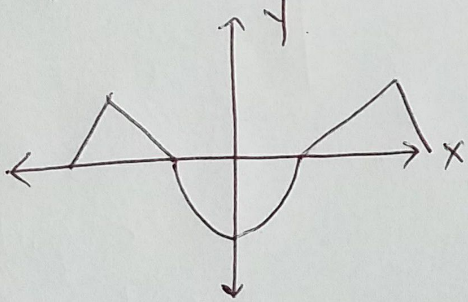
EX #2



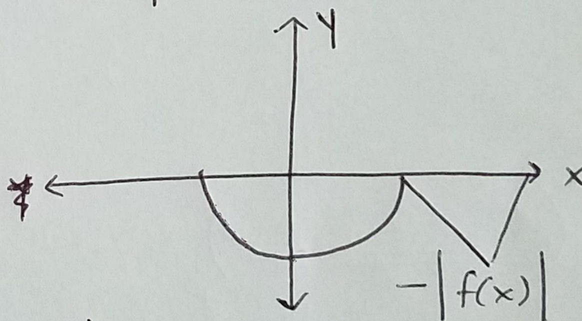
a) $y = |f(x)|$



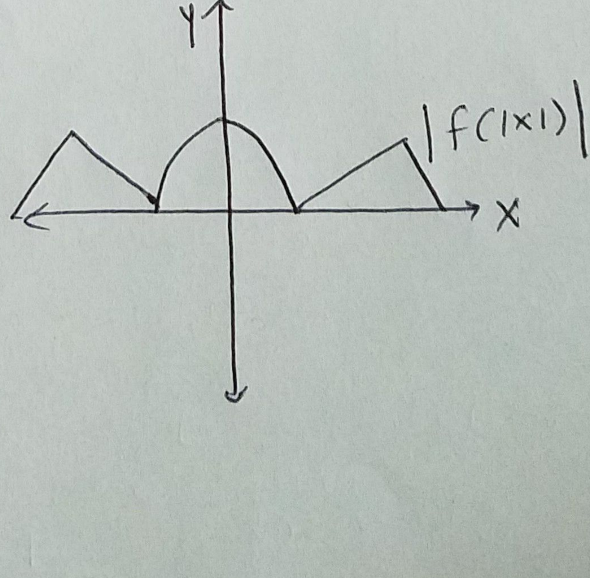
b) $y = f(|x|)$



c) $y = -|f(x)|$



d) $y = |f(|x|)|$



Stretches and Shrinks

$$y = f\left(\frac{x}{c}\right)$$

if $|c| > 1$ "a horizontal stretch by a factor of c"

if $|c| < 1$ "a horizontal shrink by a factor of c"

* Notice we are looking at $\frac{1}{c}$ (You have to take the reciprocal) *

$$y = c \cdot f(x)$$

if $|c| > 1$ "a vertical stretch by a factor of c"

if $|c| < 1$ "a vertical shrink by a factor of c"

Ex #3 Given $f(x) = 3(x-1)^2 + 4$

a) find $g(x)$, a vertical stretch of $f(x)$ by a factor of 2.

$$g(x) = 6(x-1)^2 + 4$$

b) find $h(x)$, a horizontal stretch of $f(x)$ by a factor of 2.

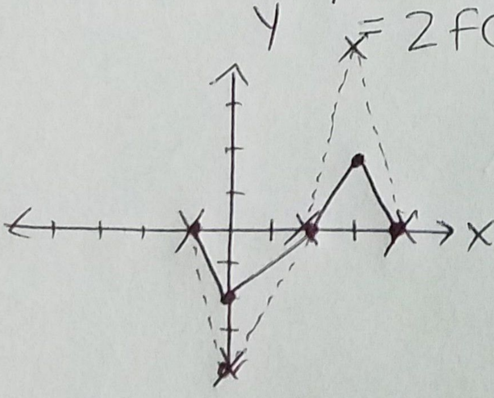
$$h(x) = 3\left(\frac{x-1}{2}\right)^2 + 4$$

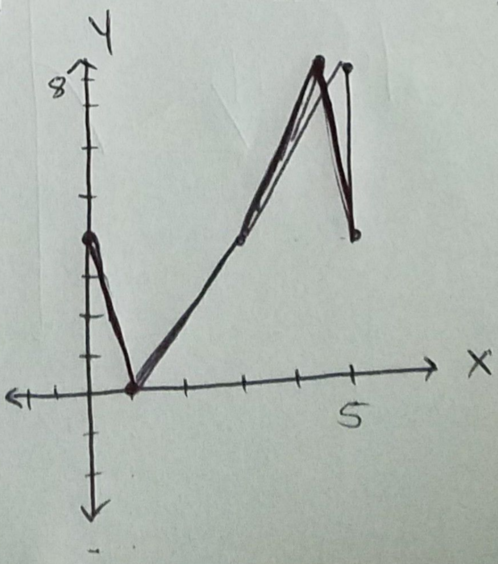
Ex #4 Given the graph of $f(x)$, sketch

$$y = 4 + 2f(x-1)$$

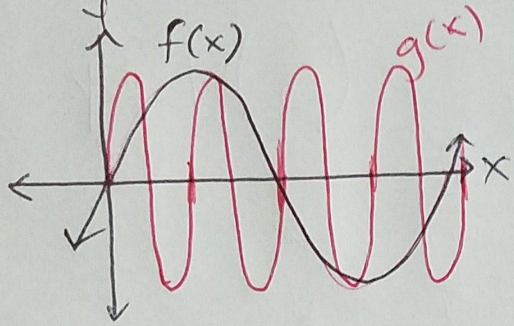
$$= 2f(x-1) + 4$$

$a f(bx+c) + d$





Ex #5



$g(x) = f(4x)$ $\quad \left|\frac{1}{4}\right| < 1$

"a horiz. sh. by a factor of $\frac{1}{4}$" **shrink**

$h(x) = f\left(\frac{1x}{2}\right)$ $\quad |2| > 1$

"a horiz. st. by a factor of 2." **stretch**