3.1/3.2/3.3

Intermediate Value Theorem

f is con't on [a,b] $\longrightarrow$ f takes on all y-values between f(a) & f(b)

$\exists\, c \in \mathbb{R} \mid f(a) < c < f(b)$

Mean Value Theorem

f is con't on [a,b]
f is diff. on (a,b) $\longrightarrow$ slope @ a point = AROC over the interval

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Rolle's Theorem

f is con't on [a,b]
f is diff. on (a,b) $\longrightarrow$ $\exists\, c \in \mathbb{R} \mid f'(c) = 0$
f(a) = f(b)

Extreme Value Theorem

f is con't on [a,b] $\longrightarrow$ $\exists$ @ least 1 min & 1 max value on [a,b]

Critical Values (candidates for extrema)

x-values where the derivative is zero or undefined <u>or</u> the endpoints of the interval

First Derivative Test

f' Δs from pos to neg @ crit. pt. $\rightarrow$ max for f
f' Δs from neg to pos @ crit. pt. $\rightarrow$ min for f

Second Derivative Test

$f'' > 0$ @ crit. value (concave up) $\rightarrow$ min for f
$f'' < 0$ @ crit. value (concave down) $\rightarrow$ max for f

Inflection Points

Location where f changes concavity
$f'' = 0 \rightarrow$ candidate for inflection point

Ex#1 Determine the intervals where $f(x)=\frac{1}{3}x^3+\frac{5}{2}x^2+6x-1$ is increasing and decreasing.

$f'(x)=x^2+5x+6$

$0=x^2+5x+6$

$0=(x+3)(x-2)$

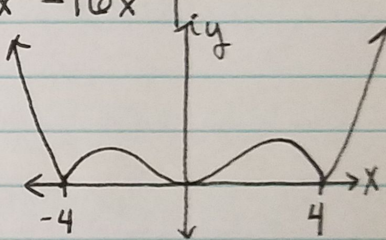
Crit. vals.: $x=-3,\ x=-2$

$f'(x)$	$+$		$-$		$+$
x		-3		-2	
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

$\boxed{\begin{array}{l} f \text{ increasing: } (-\infty,-3]\cup[-2,\infty) \\ f \text{ decreasing: } [-3,-2] \end{array}}$

Ex#2 Find all critical values for $f(x)=|x^5-16x^3|$

$$f(x)=|x^5-16x^3|=\begin{cases} -(x^5-16x^3) & x\le -4 \\ +(x^5-16x^3) & -4<x\le 0 \\ -(x^5-16x^3) & 0<x\le 4 \\ +(x^5-16x^3) & x>4 \end{cases}$$



$0=x^5-16x^3$

$0=x^3(x^2-4)$

$0=0,\pm 4$

$$f'(x)=\begin{cases} -5x^4+48x^2 & x\le -4 \\ 5x^4-48x^2 & -4<x\le 0 \\ -5x^4+48x^2 & 0<x\le 4 \\ 5x^4-48x^2 & x>4 \end{cases}$$

$5x^4-48x^2=0$

$x^2(5x^2-48)=0$

$x=0$ or $x=\pm\sqrt{48/5}$

$-5x^4+48x^2=0$

$-x^2(5x^2-48)=0$

$x=0$ or $x=\pm\sqrt{48/5}$

~~$f'(x)$ undef.~~ @ $x=\pm 4$

$\boxed{\text{crit. vals.: } x=0,\ \pm\sqrt{48/5},\ \pm 4}$

Ex #3 Find a value c such that $f'(c)=$ AROC for $f(x)=\frac{x-1}{x+2}$ on $[0,2]$ guaranteed by MVT.

$f'(c)=$ AROC

$\frac{c+2-c+1}{(c+2)^2}=\frac{\frac{1}{4}+\frac{1}{2}}{2}$

$6=\frac{3}{4}(c+2)^2$

$c=-2\pm 2\sqrt{2}$

$\boxed{c=-2+2\sqrt{2}}$