7.3 Multivariate Linear Systems & Row Operations

Vocabulary:

Coefficient Matrix: the coefficients of the system (in standard form) in a matrix.

Augmented Matrix: the coeff. matrix w/ an extra column w/ the constants

Row Echelon Form: bottom left of matrix has zeros & diagonal has ones. ↑

Reduced Row Echelon Form: but also w/ zeros on upper right

Elementary Row Operations

① add/subtract any two rows
② mult/divide any two rows
③ interchange any two rows

Goal: ones along diagonal & zeros elsewhere (except last column)

Ex #1

$$x - 2y + z = 7$$
$$3x - 5y + z = 14$$
$$2x - 2y - z = 3$$

①
$$\begin{bmatrix} 1 & -2 & 1 & | & 7 \\ 3 & -5 & 1 & | & 14 \\ 2 & -2 & -1 & | & 3 \end{bmatrix}$$

$-3R_1 + R_2$ ②
$$\begin{bmatrix} 1 & -2 & 1 & | & 7 \\ 0 & 1 & -2 & | & -7 \\ 2 & -2 & -1 & | & 3 \end{bmatrix}$$

③
$$\begin{bmatrix} 1 & -2 & 1 & | & 7 \\ 0 & 1 & -2 & | & -7 \\ 0 & 2 & -3 & | & -11 \end{bmatrix}$$

$-2R_1 + R_3$

$-2R_2 + R_3$ ④
$$\begin{bmatrix} 1 & -2 & 1 & | & 7 \\ 0 & 1 & -2 & | & -7 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$R_1 + 2R_2$ ⑤
$$\begin{bmatrix} 1 & 0 & -3 & | & -7 \\ 0 & 1 & -2 & | & -7 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$3R_3 + R_1$ ⑥
$$\begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & -2 & | & -7 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$3R_3 + R_2$ ⑦
$$\begin{bmatrix} 1 & 0 & 0 & | & 2 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$$\boxed{(2, -1, 3)}$$

Ex #2

$x - y + 2z = -3$

$2x + y - z = 0$

$-x + 2y - 3z = 7$

$$\begin{bmatrix} 1 & -1 & 2 & | & -3 \\ 2 & 1 & -1 & | & 0 \\ -1 & 2 & -3 & | & 7 \end{bmatrix}$$

$-2R_1 + R_2$

$$\begin{bmatrix} 1 & -1 & 2 & | & -3 \\ 0 & 3 & -5 & | & 6 \\ -1 & 2 & -3 & | & 7 \end{bmatrix}$$

$R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & 2 & | & -3 \\ 0 & 3 & -5 & | & 6 \\ 0 & 1 & -1 & | & 4 \end{bmatrix}$$

R_{23}

$$\begin{bmatrix} 1 & -1 & 2 & | & -3 \\ 0 & 1 & -1 & | & 4 \\ 0 & 3 & -5 & | & 6 \end{bmatrix}$$

$3R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & 2 & | & -3 \\ 0 & 1 & -1 & | & 4 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 1 & | & 1 \\ 0 & 1 & -1 & | & 4 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$-R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & -1 & | & 4 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 7 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$(-2, 7, 3)$

Inverse Matrices

We will have a coefficient matrix, a variable matrix, & a constant matrix.

$$x - y + 2z = -3$$
$$2x + y - z = 0$$
$$-x + 2y - 3z = 7$$

$$\Longrightarrow \quad \overset{A}{\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{B}{\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}}$$

$$AX = B$$

$$\cancel{A^{-1}A}X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

Ex #3

$$3x - 2y = 0$$
$$-x + y = 5$$

$$\underset{A}{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \underset{B}{\begin{bmatrix} 0 \\ 5 \end{bmatrix}}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \qquad \boxed{(10, 15)}$$

Ex #4

$$3x - 3y + 6z = 20$$
$$x - 3y + 10z = 40$$
$$-x + 3y - 5z = 30$$

$$\boxed{\left(18, \frac{118}{3}, 14\right)}$$