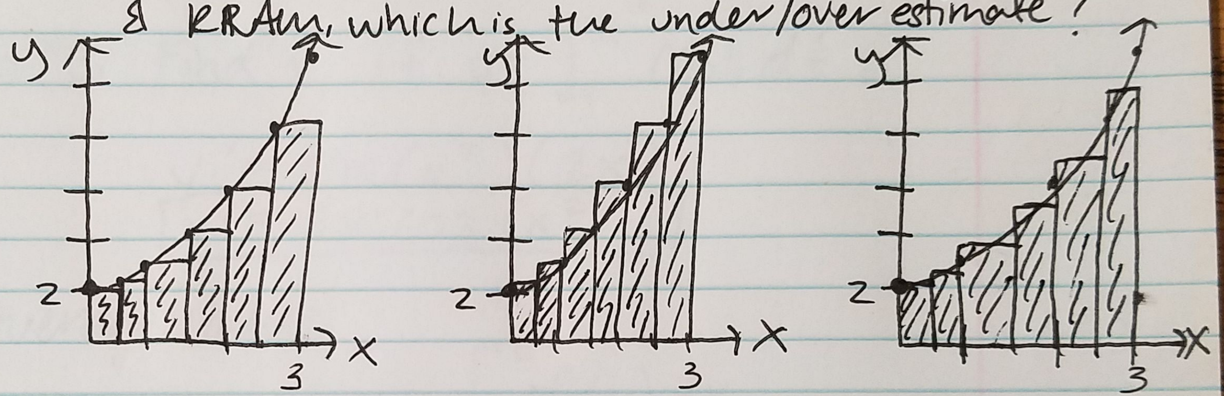
4.5/4.6

Riemann Sums

Riemann came up with the Rectangular Approximation Method (RAm) that uses rectangles to approximate the area under the curve.

Ex#1 Consider $y = x^2 + 2$ on $[0, 3]$. Evaluate LRAM, RRAM & MRAM w/ 6 subintervals. Between LRAM & RRAM, which is the under/over estimate?



LRAM RRAM MRAM

LRAM $= 0.5(2 + 2.25 + 3 + 4.25 + 6 + 8.25)$

$\boxed{\text{LRAM} = 12.875}$

under

RRAM $= 0.5(2.25 + 3 + 4.25 + 6 + 8.25 + 11)$

$\boxed{\text{RRAM} = 17.375}$

over

MRAM $= 0.5(2.0625 + 2.5625 + 3.5625 + 5.0625 + 7.0625 + 9.5625)$

$\boxed{\text{MRAM} = 14.9375}$

Fundamental Theorem of Calculus

$$y' = \frac{d}{dx} \int_a^x f(t)\,dt = f(x)$$

a = constant & x has nothing else.

$$y' = \frac{d}{du}\int_a^u f(t)\,dt\ \frac{du}{dx} = f(u)\ \frac{du}{dx}$$

u is a function of x & a is a constant

Ex #2 Find y' if $y = \int_{-\pi}^{x} \cos t\,dt$

$$\boxed{y' = \cos x}$$

Ex #3 Find y' if $y = \int_1^{x^2} \cos t\,dt$.

$$y' = \cos(x^2)(2x)$$

$$\boxed{y' = 2x\cos(x^2)}$$

Average Value

$$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Ex #4 Find the average value of $f(x) = 2\sec^2 x$ over $[-\pi/4, \pi/4]$.

$$av(f) = \frac{1}{\frac{\pi}{4}+\frac{\pi}{4}}\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx$$

$$= \frac{2}{\pi}\,2\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx$$

$$= \frac{4}{\pi}\Big[\tan x\Big]_{-\pi/4}^{\pi/4}$$

$$= \frac{4}{\pi}\big(1-(-1)\big)$$

$$\boxed{av(f) = \frac{8}{\pi}}$$