Arroyo P.6 3/2/22

A52: Solve Square Roots and Graph Cube Roots

25.2 Solve Square Root Equations & Graph Cube Root Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Square Root Equations

Isolate the radical/square root, then square both sides of the equation.

Check your answers for extraneous solutions

→ your math is perfect, but doesn't work when you plug back in

Ex #1 Solve and check for extraneous solutions.

a) $\sqrt{x-3} + 4 = 9$

$-4 \quad -4$

$(\sqrt{x-3})^2 = (5)^2$

$x - 3 = 25$

$+3 \quad +3$

$\boxed{x = 28}$

Check

$\sqrt{28-3} + 4 = 9$

$\sqrt{25} + 4 = 9$

$5 + 4 = 9$

$9 = 9$ ✓

b) $x = (x+1)^{\frac{1}{2}} + 5$

$x = \sqrt{x+1} + 5$

$-5 \quad -5$

$(x-5)^2 = (\sqrt{x+1})^2$

$(x-5)(x-5) = x+1$

	x	-5
x	x^2	$-5x$
-5	$-5x$	$+25$

$x^2 - 10x + 25 = x + 1$

$-x \quad -1 \quad -x \quad -1$

$x^2 - 11x + 24 = 0$

$(x-8)(x-3) = 0$

$\boxed{x = 8}$ $\quad x = 3$

ext.

Check

$8 = (8+1)^{1/2} + 5$

$8 = \sqrt{9} + 5$

$8 = 3 + 5$ ✓

$3 = (3+1)^{1/2} + 5$

$3 = \sqrt{4} + 5$

$3 = 2 + 5$ ✗

c) $2 - \sqrt{x+1} = -5$

$-2 \quad -2$

$-\sqrt{x+1} = -7$

$\div -1 \quad \div -1$

$(\sqrt{x+1})^2 = (7)^2$

$x + 1 = 49$

$-1 \quad -1$

$\boxed{x = 48}$

Check

$2 - \sqrt{48-1} = -5$

$2 - \sqrt{49} = -5$

$2 - 7 = -5$ ✓

d) $(\sqrt{x+4})^2 = (x-8)^2$

$x + 4 = x^2 - 16x + 64$

$-x \quad -4 \quad -x \quad -4$

$0 = x^2 - 17x + 60$

$0 = (x-12)(x-5)$

$\boxed{x = 12}$ $\quad x = 5$

ext.

Check

$\sqrt{12+4} = 12 - 8$

$\sqrt{16} = 4$ ✓

$\sqrt{5+4} = 5 - 8$

$\sqrt{9} = -3$ ✗

e) $(x+6)^{\frac{1}{2}} = -x$

$(\sqrt{x+6})^2 = (-x)^2$

$x + 6 = x^2$

$-x \quad -6 \quad -x \quad -6$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3 \quad \boxed{x = -2}$

ext.

Check

$\sqrt{3+6} = -3$

$3 = -3$ ✗

$\sqrt{-2+6} = 2$

$\sqrt{4} = 2$

$2 = 2$ ✓

f) $(x+4)^{\frac{1}{2}} + 1 = 0$

$-1 \quad -1$

$\sqrt{x+4} = -1$ ← we know the ans. is $\boxed{\text{No solution}}$

$(\sqrt{x+4})^2 = (-1)^2$

$x + 4 = 1$

$-4 \quad -4$

$x = -3$

ext.

$\boxed{\text{No solution}}$

check

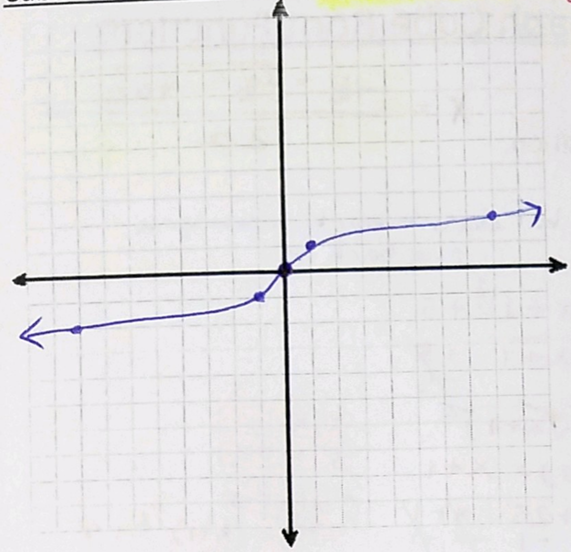
$\sqrt{-3+4} + 1 = 0$

$\sqrt{1} + 1 = 0$

$2 = 0$

$\neq$

Cube Root Parent Graph: $f(x) = \sqrt[3]{x}$ Negatives inside $\sqrt[3]{x}$ are OK



x	y
0	0
1	1
8	2
-8	-2
-1	-1

$f(x) = \sqrt[3]{x}$ has the points:

(-8 , -2) (-1 , -1) (0 , 0) (1 , 1) (8 , 2)

Cube Root Transformations

- negative: reflection over x-axis
- $|a|<1$ vertical shrink
- $|a|>1$ vertical stretch

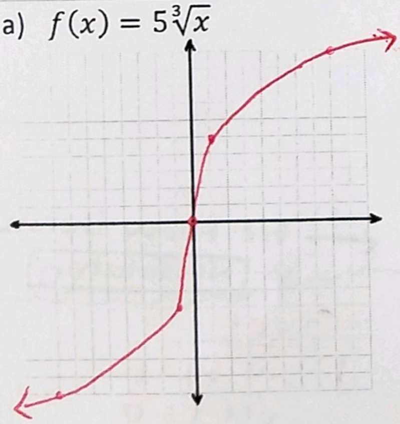
mult the y-vals by "a"

$$f(x) = a\sqrt[3]{x-c} + d$$

- +d up
- −d down
- +c left
- −c right

Ex #2 Graph the function, then state the domain and range.

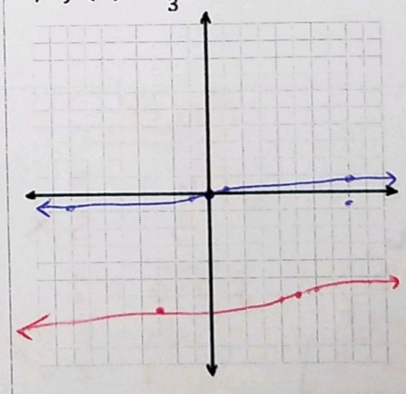
a) $f(x) = 5\sqrt[3]{x}$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

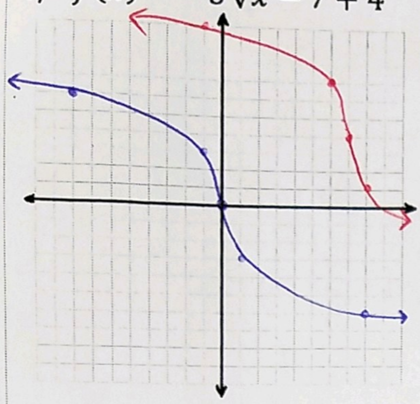
b) $f(x) = \frac{1}{3}\sqrt[3]{x-5} - 6$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

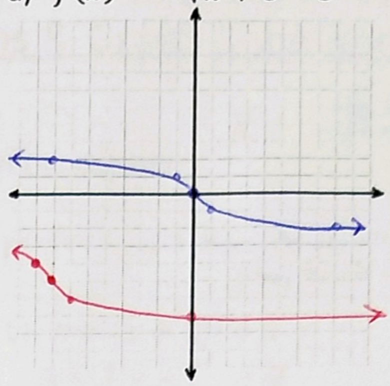
c) $f(x) = -3\sqrt[3]{x-7} + 4$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

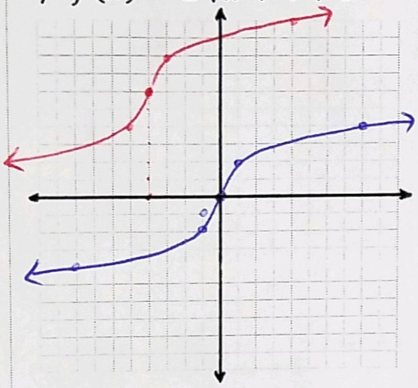
d) $f(x) = -\sqrt[3]{x+8} - 5$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

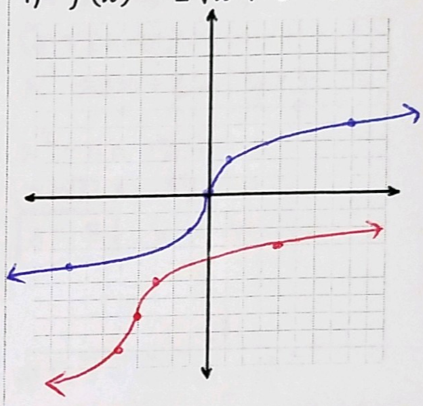
e) $f(x) = 2\sqrt[3]{x+4} + 6$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

f) $f(x) = 2\sqrt[3]{x+4} - 7$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

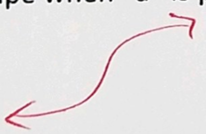
<u>Key Features of Cube Root Graphs</u>

What are the domain and range of cube root functions?

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

What is the shape when "a" is positive?



What is the shape when "a" is negative?