2.7 Solving Equations in One Variable

① Factor the denominator
② Multiply every term by the LCD
③ Solve the resulting equation
④ Check for extraneous solutions.

Ex #1 Solve $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$.

LCD: $5x(x+2)$

$$5x(x+2)\left(\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}\right)$$

$$\frac{3}{x+2}\left(\frac{5x(x+2)}{1}\right) - \frac{1}{x}\left(\frac{5x(x+2)}{1}\right) = \frac{1}{5x}\left(\frac{5x(x+2)}{1}\right)$$

$$3(5x) - 5(x+2) = (x+2)$$

$$15x - 5x - 10 = x + 2$$

$$10x - 10 = x + 2$$

$$9x = 12$$

$$x = \frac{4}{3}$$

Check: $\frac{3}{\frac{4}{3}+\frac{6}{3}} - \frac{1}{\frac{4}{3}} = \frac{1}{5(\frac{4}{3})}$

$$\frac{3}{\frac{10}{3}} - \frac{1}{\frac{4}{3}} = \frac{1}{\frac{20}{3}}$$

$$3\left(\frac{3}{10}\right) - 1\left(\frac{3}{4}\right) = 1\left(\frac{3}{20}\right)$$

$$\frac{9}{10} - \frac{3}{4} = \frac{3}{20}$$

$$\frac{18}{20} - \frac{15}{20} = \frac{3}{20}$$

$$\frac{3}{20} = \frac{3}{20} \checkmark$$

$\therefore x = \frac{4}{3}$ is a solution

Ex #2 Solve $x + 1 = \frac{72}{x}$. LCD: x

$$x^2 + x = 72$$

$$x^2 + x - 72 = 0$$

$$(x+9)(x-8) = 0$$

$$x = -9,\ x = 8$$

Check: $-9 + 1 = \frac{72}{-9}$ $\quad 8 + 1 = \frac{72}{8}$

$-8 = -8 \checkmark$ $\quad 9 = 9 \checkmark$

$\therefore x = -9$, $x = 8$ are solutions

Ex #3 Solve $\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{x^2-6x+8}$

$\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{(x-4)(x-2)}$

LCD: $(x-2)(x-4)$

$x(x-4) + (x-2) = 2$

$x^2 - 4x + x - 2 - 2 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ & $x = -1$

Check: $\frac{4}{4-2} + \frac{1}{4-4} = \frac{2}{4^2-6(4)+8}$

undefined

$\frac{-1}{-1-2} + \frac{1}{-1-4} = \frac{2}{(-1)^2-6(-1)+8}$

$\frac{1}{3} + \frac{1}{-5} = \frac{2}{15}$

$\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$

$\frac{2}{15} = \frac{2}{15}$ ✓

$x = 4$ is an extraneous soln

$x = -1$ is a solution