9.5 Series

Vocabulary

Series: The sum of a list of terms / sum of a sequence.

Summation / Sigma Notation: The form used to represent the sum of a sequence.

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n$$

"the sum of a_k from k=1 to n"

Finite series: A series with an end.

Infinite Series: A series that goes on forever.

Partial Sum: A peice of an infinite series (can also consider it a finite series)

Ex# 1 Evaluate the following:

a) $\sum_{k=1}^{5} 3k = 45$

b) $\sum_{k=5}^{8} k^2 = 174$

c) $\sum_{k=0}^{12} \cos(k\pi) = 1$

Sum of a Finite Arithmetic Sequence (which is a series)

$$S_n = \sum_{k=1}^{n} a_k = n\left(\frac{a_1 + a_n}{2}\right) = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

Sum of a Finite Geometric Sequence

$$S_n = \sum_{k=1}^{n} a_k = a_1\left(\frac{1-r^n}{1-r}\right)$$

Sum of an Infinite Geometric Sequence

$a_n = a_1(r)^{n-1}$

$$S = \sum_{k=1}^{\infty} a_k = \frac{a_1}{1-r} \quad \text{iff} \quad |r| < 1$$

Ex #2 Determine the sum of the following:

a) $4 - 4/3 + 4/9 - 4/27 + \cdots + 4(-1/3)^{10}$

$10 = n-1$

$11 = n$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $\quad a_1 = 4 \quad r = -1/3 \quad n = 11$

$S_{11} = 4\left(\frac{1-(-1/3)^{11}}{1+1/3}\right)$

$S_{11} \approx 3.000016935\ldots$

$S_{11} \approx 3.000$

b) How many seats are in a corner section of a stadium that has 8 seats in the front row & 2 additional seats in each row preceeding it. There are 24 seats in the top row.

$a_1 = 8 \quad a_n = 24 \quad d = 2 \quad n = ?$

$a_n = a_1 + (n-1)d$

$24 = 8 + (n-1)2$

$n = 9 \qquad S_9 = 144$ seats

c) $\sum_{n=0}^{\infty} \left(-\frac{4}{5}\right)^n = \sum_{n=1}^{\infty} \left(-\frac{4}{5}\right)^{n-1}$

$a_1 = 1 \quad r = -4/5 \qquad S = 5/9$

d) $\sum_{n=1}^{\infty} 3(0.75)^{n-1}$

$a_1 = 3 \quad r = 0.75 \qquad S = 12$

e) $\sum_{n=0}^{\infty} \left(\frac{\pi}{2}\right)^n \qquad r = \frac{\pi}{2} > 1$

The series diverges.

f) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$

$a_1 = 1 \quad r = \frac{1}{2} \qquad S = 2$

Ex#3 Determine if the series appears to converge by using the first 5 partial sums.

a) $0.1 + 0.01 + 0.001 + 0.0001 + \cdots$

$S_1 = 0.1 \quad S_2 = 0.11 \quad S_3 = 0.111 \quad S_4 = 0.1111 \quad S_5 = 0.11111$

S appears to be $0.\overline{1111} = 1/9$

b) $10 + 20 + 30 + 40 + \cdots$

$S_1 = 10 \quad S_2 = 30 \quad S_3 = 60 \quad S_4 = 100 \quad S_5 = 150$

appears to diverge

c) $1 - 1 + 1 - 1 + \cdots$

$S_1 = 1 \quad S_2 = 0 \quad S_3 = 1 \quad S_4 = 0 \quad S_5 = 1$ diverges

Ex#4 Convert the following decimals to fractions

a) $0.\overline{234} = 0.234 + 0.000234 + 0.000000234 + \cdots$

$= \sum_{n=1}^{\infty} 0.234(0.001)^{n-1}$

$= \frac{0.234}{1 - 0.001}$

$= \frac{0.234}{0.999} = \frac{234}{999} = \boxed{\frac{26}{111}} = 0.\overline{234}$

b) $1.41\overline{41} = 1 + .41\overline{41} = 1 + \frac{41}{99} = \boxed{\frac{140}{99}}$