9.4 Sequences

Vocabulary

Sequence: A list of numbers in an ordered progression.

ex: 2, 4, 6, 8, 10

ex: 2, 4, 8, 16, 32, ..., 2^n, ...

ex: $\{\frac{1}{n} \mid n = 1, 2, 3, \ldots\}$

ex: $\{a_1, a_2, a_3, \ldots, a_n, \ldots\}$

n^{th} term: A rule/formula, in terms of n, that you can use to determine any # in the sequence.

Ex #1 Find the first 6 terms & the 200^{th} term of $a_n = n^2 + 7$.

Explicit rule

Explicitly defined sequence

$a_1 = 8$

$a_2 = 11$

$a_3 = 16$

$a_4 = 23$

$a_5 = 32$

$a_6 = 43$

$a_{200} = 40{,}007$

Ex #2 Determine the first 6 terms & the 20^{th} term of $b_n = b_{n-1} + 3$ and $b_1 = 2$. (recursive rule)

$b_1 = 2$

$b_2 = 5$

$b_3 = 8$

$b_4 = 11$

$b_5 = 14$

$b_6 = 17$

$\vdots$

$b_{20} = 59$

Limits & Convergence

If $\lim_{n\to\infty} a_n = L$ & L is a finite number then the sequence converges to L.

If $L = \pm\infty$ or is undefined / DNE, then the sequence diverges.

Ex #3 Determine what the sequence converges to, if it converges.

a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$a_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$

The sequence $a_n = \frac{1}{n}$ converges to 0.

b) $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots$

$a_n = \frac{n+1}{n}$

$\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1} = 1$

The sequence $a_n = \frac{n+1}{n}$ converges to 1.

c) $2, 4, 6, 8, 10, \ldots$

$a_n = 2n$

$\lim_{n\to\infty} 2n = \infty$

The sequence $a_n = 2n$ diverges to ∞.

d) $-1, 1, -1, 1, -1, \ldots$

$a_n = (-1)^n$

$\lim_{n\to\infty} (-1)^n$ DNE

The sequence $a_n = (-1)^n$ diverges since $\lim_{n\to\infty} (-1)^n$ DNE.

e) $\left\{\frac{n^3+2}{n^2+n}\right\}$

$\lim_{n\to\infty} \frac{n^3+2}{n^2+n} = \lim_{n\to\infty} \frac{1+\frac{2}{n^3}}{\frac{1}{n}+\frac{1}{n^2}} = \text{undefined}$

The sequence $\left\{\frac{n^3+2}{n^2+n}\right\}$ diverges since limit is undef.

Arithmetic Sequences

Add or subtract a constant, d.

d = common difference $= a_n - a_{n-1}$

Explicit Rule: $a_n = a_1 + (n-1)d$

Recursive Rule: $a_n = a_{n-1} + d$ (include a_1)

Geometric Sequences

multiply by a common ratio, r.

r = common ratio $= a_{n+1}/a_n$

Explicit rule: $a_n = a_1 (r)^{n-1}$

Recursive Rule: $a_n = a_{n-1} \cdot r$ (include a_1)

Ex #4 Determine the common difference/ ratio, the 10^{th} term, the explicit rule, & the recursive rule.

a) $-6, -2, 2, 6, 10, \ldots$

$\boxed{d = 4}$

$a_n = -6 + (n-1)4$

$\boxed{a_n = 4n - 10}$

$\boxed{a_{10} = 30}$ $\boxed{a_n = a_{n-1} + 4 \; ; \; a_1 = -6}$

b) $3, 6, 12, 24, 48, \ldots$

$\boxed{r = 2}$ $\boxed{a_n = 3(2)^{n-1}}$ $\boxed{a_{10} = 1536}$

$\boxed{a_n = 2a_{n-1} \; ; \; a_1 = 3}$

Ex #5 The 2^{nd} & 5^{th} terms of a sequence are 3 & 24, respectively. Find the explicit & recursive formulas if

a) arithmetic

$a_2 = 3$

$a_5 = 24$

$a_n = a_1 + (n-1)d$

$a_5 - a_2 = [\cancel{a_1} + (4)d] - [\cancel{a_1} + (1)d]$

$24 - 3 = 3d$

$21 = 3d$

$d = 7$:)

$a_2 = a_1 + (2-1)7$

$3 = a_1 + 7$

$a_1 = -4$:)

$a_n = -4 + (n-1)7$

$\boxed{a_n = 7n - 11}$

$\boxed{a_n = a_{n-1} + 7 \quad a_1 = -4}$

b) geometric

$a_2 = 3$

$a_5 = 24$

$a_n = a_1(r)^{n-1}$

$\frac{a_5}{a_2} = \frac{\cancel{a_1}(r)^4}{\cancel{a_1}(r)^1}$

$\frac{24}{3} = r^3$

$8 = r^3$

$r = 2$:)

$a_2 = a_1(2)^{2-1}$

$3 = 2a_1$

$a_1 = 3/2$:)

$\boxed{a_n = \frac{3}{2}(2)^{n-1}}$

$\boxed{a_n = 2a_{n-1} \quad a_1 = \frac{3}{2}}$