1.14 / 1.16 / 1.17

Limits

Ex#1 $\lim_{x\to\infty} \frac{6x-8}{8-2x} = \lim_{x\to\infty} \frac{6-\frac{8}{x}}{\frac{8}{x}-2}$

$= \frac{\lim_{x\to\infty} 6 - \lim_{x\to\infty} \frac{8}{x}}{\lim_{x\to\infty} \frac{8}{x} - \lim_{x\to\infty} 2}$

$= \frac{6-0}{0-2}$

$\boxed{\lim_{x\to\infty} \frac{6x-8}{8-2x} = -3}$

Ex#2 $\lim_{x\to\infty} \frac{7x^2+2}{3x-1} = \lim_{x\to\infty} \frac{7x+\frac{2}{x}}{3-\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{7x}{3}$

$\boxed{\lim_{x\to\infty} \frac{7x^2+2}{3x-1} = \infty}$

Ex#3 $\lim_{x\to\infty} \frac{\sqrt[3]{x^3+1}}{x+2} \approx \lim_{x\to\infty} \frac{\sqrt[3]{x^3}}{x}$

$= \lim_{x\to\infty} \frac{x}{x}$

$\boxed{\lim_{x\to\infty} \frac{\sqrt[3]{x^3+1}}{x+2} = 1}$

Ex#4 $\lim_{x\to 4} \frac{2x+1}{x-4} = \lim_{x\to 4} \left(2+\frac{9}{x-4}\right)$

$$\begin{array}{r} 2+\frac{9}{x-4} \\ x-4\,\overline{)\,2x+1} \\ -(2x-8) \\ \hline 9 \end{array}$$

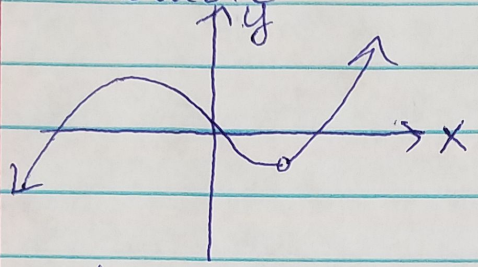
$\lim_{x\to 4} \left(\frac{2x+1}{x-4}\right)$ DNE because the function has a VA at $x=4$ & is a translation of $\frac{1}{x}=y$.

Ex#5 $\lim_{x\to -3} \frac{2x^3}{x^2-9} \approx \begin{cases} \lim_{x\to -3.0001} \frac{2x^3}{x^2-9} \approx -90,000 \\ \lim_{x\to -2.9999} \frac{2x^3}{x^2-9} \approx 90,000 \end{cases}$

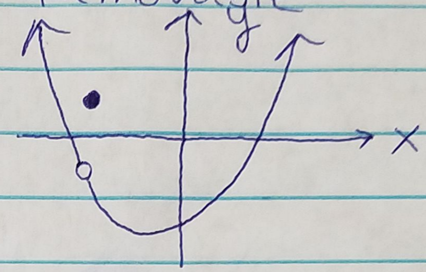
$\lim_{x\to -3} \frac{2x^3}{x^2-9}$ DNE since $\lim_{x\to -3^-} \frac{2x^3}{x^2-9} \neq \lim_{x\to -3^+} \frac{2x^3}{x^2-9}$.

Types of Discontinuities

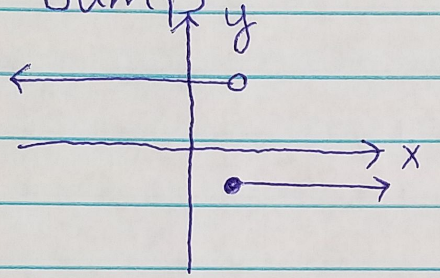
Removable



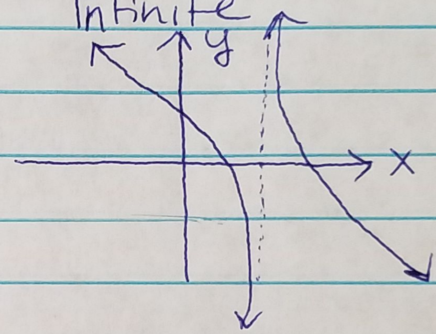
Removable



Jump



Infinite



Continuity

A function is continuous if $\lim_{x \to a} f(x) = f(a)$.
The same is true for one-sided limits at endpoints.