2.12/2.13

- Draw a picture / diagram (if possible)
- Identify knowns & unknowns
- Set up an equation with the two necessary variables.
    - Watch out for extra variables & be careful w/ constant values vs values at a particular time.
- Take the derivative implicitly WRT, t.
- Sub in values & don't forget units!

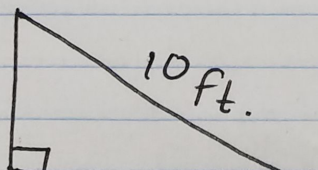
Ex #1 If $xy = -3$ and $\frac{dx}{dt} = 1$, find $\frac{dy}{dt}$ when $x = 6$.

$$xy = -3$$
$$\frac{dx}{dt}y + x\frac{dy}{dt} = 0$$
$$1\left(-\tfrac{1}{2}\right) + 6\frac{dy}{dt} = 0$$
$$6\frac{dy}{dt} = \frac{1}{2}$$
$$\boxed{\frac{dy}{dt} = \frac{1}{12}}$$

★ $6y = -3$
★ $y = -\frac{1}{2}$ ★

Ex #2 A 10 ft. ladder placed on a wall is sliding along the ground at a rate of 4 ft/sec. What is the rate at which the ladder is sliding down the wall when the ladder is 8 ft away from the base of the wall.

$\downarrow$ $\frac{dy}{dt} = ?$



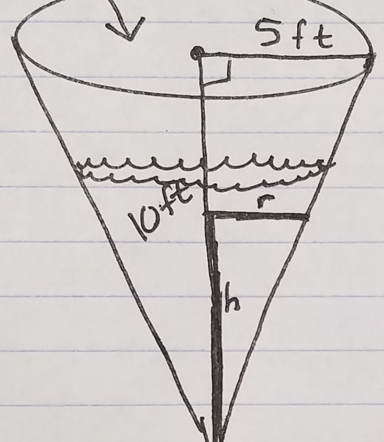
$\rightarrow \frac{dx}{dt} = 4$ ft/sec

when $x = 8$ ft, $y = 6$ ft

$$x^2 + y^2 = 100$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(8)(4) + 2(6)\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = \frac{-64}{12}$$
$$\frac{dy}{dt} = -\frac{16}{3} \text{ ft/sec}$$

Ex #3 Water is pouring into a conical tank at rate of 9 cubic feet per minute. The height of the tank is 10ft and the radius at the top is 5ft. At what rate is the height of the water increasing when the radius is 3 ft?

$$\frac{dV}{dt} = 9\ \frac{ft^3}{min}$$



$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$$

$$9 = \frac{\pi}{12}(3(36))\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{\pi}\ ft/min}$$

★ $\frac{r}{5} = \frac{h}{10}$

$r = \frac{h}{2}$

$3 = \frac{h}{2}$

$6 = h$