Notes: 27.1/27.2/28.2 Rational Word Problems & Transformations

The Summer Camp Problem

A summer camp is open for 5 days and wants to charge its campers exactly enough to cover all the costs of running the camp. It costs the camp $2400 per day to pay for its rent, employees, insurance, and equipment.

a. What is the total cost to run the camp for the 5 days?

$$2400(5) = \boxed{\$12,000}$$

b. Write an equation that represents how much each camper should be charged.

$x =$ total # of campers

$y = \$$ charged per camper

$$\boxed{\frac{12000}{x} = y}$$

c. The campers will eat three meals a day and each meal costs $3. Adjust your equation (from part b) to include the cost of meals.

$$3(3)(5) = \$45 \Big/ \substack{camper \\ week}$$

$$\boxed{\frac{12000 + 45x}{x} = y}$$

d. The camp is allowing 30 campers in for free. Adjust your equation (from part c) to include the campers that are going for free.
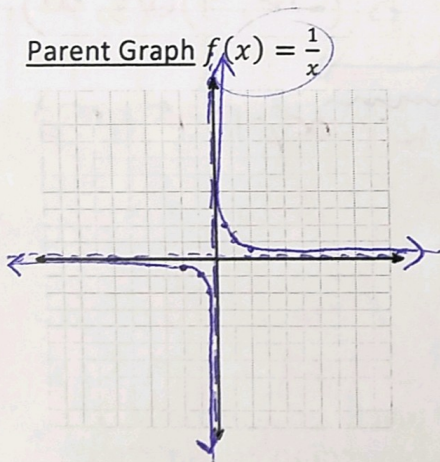
$$\boxed{\frac{12000 + 45x}{x - 30} = y}$$

e. How much would each paying camper have to pay if a total of 500 campers end up going?

$$\frac{(12000 + 45(500))}{(500 - 30)} = y$$

$$y = 73.40425532$$

$$\boxed{y = \$73.41}$$

Parent Graph $f(x) = \frac{1}{x}$

$f(x) = \frac{1}{x}$ has the points:

$(\underline{-2}, \underline{-\tfrac{1}{2}})$ $(\underline{-1}, \underline{-1})$ $(\underline{-\tfrac{1}{2}}, \underline{-2})$

$(\underline{\tfrac{1}{2}}, \underline{2})$ $(\underline{1}, \underline{1})$ $(\underline{2}, \underline{\tfrac{1}{2}})$

Rational Transformations

neg: reflection over x-axis
$|a| < 1$ vertical shrink
$|a| > 1$ vertical stretch

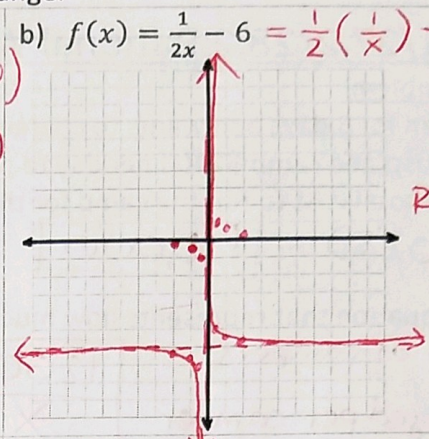$f(x) = \dfrac{a}{x - c} + d$

+d up
−d down

+c left
−c right

$$\left(-2, -\tfrac{1}{2}\right) \left(-1, -1\right) \left(-\tfrac{1}{2}, -2\right) \quad \left(\tfrac{1}{2}, 2\right) \left(1, 1\right) \left(2, \tfrac{1}{2}\right)$$

Ex #1: Graph the function, then state the domain and range.

a) $f(x) = -\dfrac{5}{x}$

$D: (-\infty, 0) \cup (0, \infty)$

$R: (-\infty, 0) \cup (0, \infty)$

U

b) $f(x) = \dfrac{1}{2x} - 6 = \tfrac{1}{2}\left(\tfrac{1}{x}\right) - 6 = \dfrac{\frac{1}{2}}{x} - 6$

$D: (-\infty, 0) \cup (0, \infty)$

$R: (-\infty, -6) \cup (-6, \infty)$

c) $f(x) = \dfrac{1}{x-7} - 5$

$D: (-\infty, 7) \cup (7, \infty)$

$R: (-\infty, -5) \cup (-5, \infty)$

d) $f(x) = -\dfrac{4}{x+5} + 2$

$D: (-\infty, -5) \cup (-5, \infty)$

$R: (-\infty, 2) \cup (2, \infty)$

e) $f(x) = 3 + \dfrac{4}{x-2} = \dfrac{4}{x-2} + 3$

$D: (-\infty, 2) \cup (2, \infty)$

$R: (-\infty, 3) \cup (3, \infty)$

f) $f(x) = \dfrac{6}{3x-9} - 1 = \tfrac{1}{3}\left(\dfrac{6}{x-3}\right) - 1 = \dfrac{6/3}{x-3} - 1$

$= \dfrac{2}{x-3} - 1$

$D: (-\infty, 3) \cup (3, \infty)$

$R: (-\infty, -1) \cup (-1, \infty)$
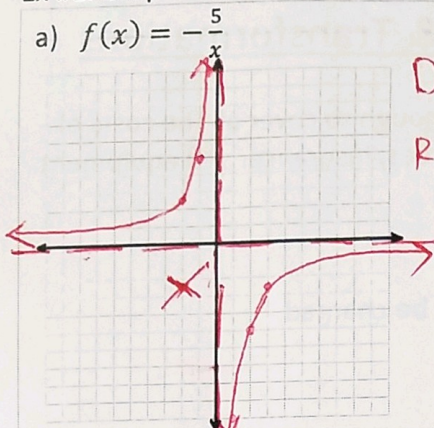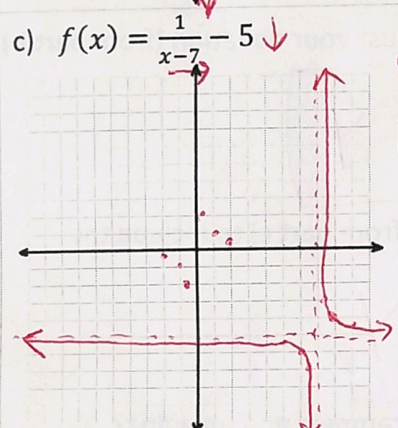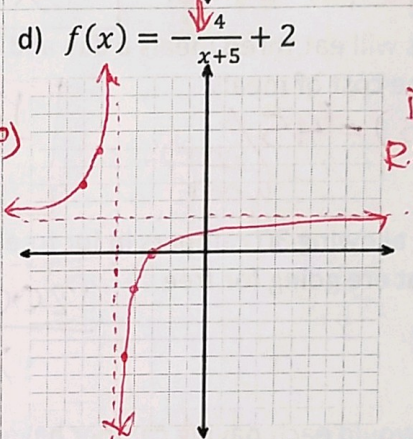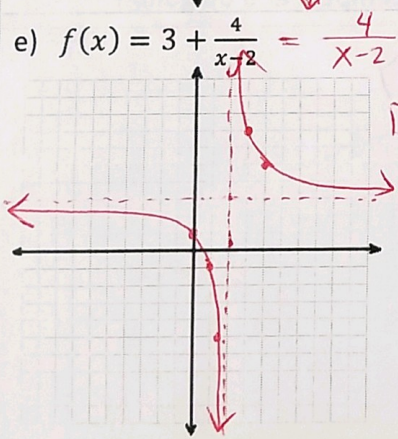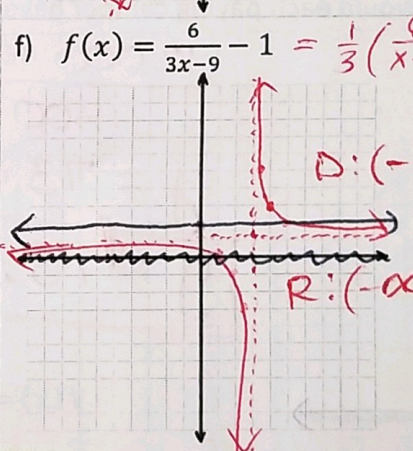
~~Key Features of Square Root Graphs~~ Rational

How is the horizontal translation (left/right) related to the domain and the vertical asymptote?

Left/right movement tells us where the vertical asymptote is at and what the D should be.

How is the vertical translation (up/down) related to the range and horizontal asymptote?

Up/down movement tells us where the horiz. asymptote is at and what the R should be.

What do you do if there is a number multiplying the x in the denominator?

Factor it out & multiply it by the numerator!

A55: Graphing Rational Functions and Word Problems