A 31: pg 289 # 1–14, 17, 19–21

18.1/18.2 Graphing Polynomials & The Rational Root Theorem

Graphing Polynomials Summary

Leading Coefficient – tells you how the right ends
- positive – end up on right
- negative – end down on right

Degree – tells you how left ends & max # of x-int
- even – same as right
- odd – opposite of right

Constant – tells you y-int

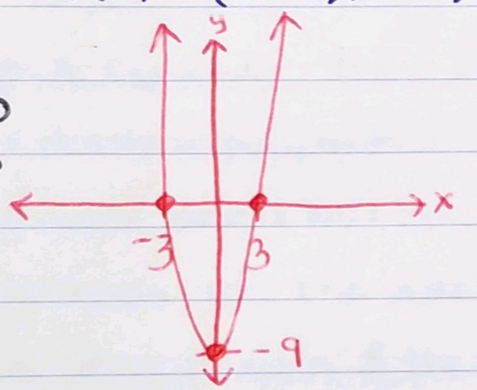
Factors – tells you the x-ints.

Ex #1 $f(x) = x^2 - 9$

$f(x) = (x+3)(x-3)$

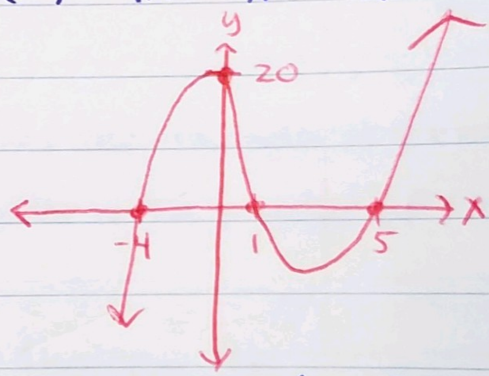
$x + 3 = 0$

$x = -3$



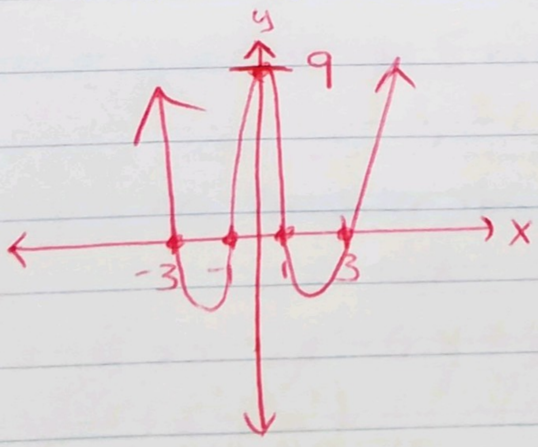
Ex #2 $f(x) = x^3 - 2x^2 - 19x + 20$

$f(x) = (x+4)(x-1)(x-5)$



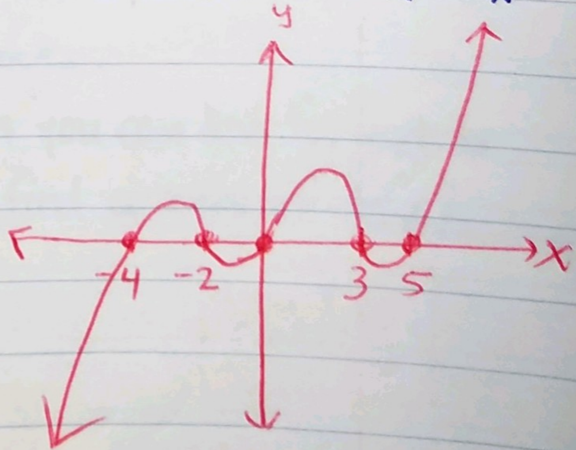
Ex #3 $f(x) = x^4 - 10x^2 + 9$

$f(x) = (x+3)(x-3)(x+1)(x-1)$



Ex #4 $f(x) = x^5 - 2x^4 - 25x^3 + 26x^2 + 120x$

$f(x) = x(x-5)(x-3)(x+2)(x+4)$



Rational Root Theorem

If a polynomial has integer coefficients, then every rational root has the form $\frac{p}{q}$, where p is a factor of the constant & q is a factor of the leading coeff.

Ex #5 Write the POSSIBLE rational roots of:

a) $f(x) = 2x^3 - 2x^2 - 5x + 6$

$p = \pm 1, \pm 3, \pm 2, \pm 6$

$q = \pm 1, \pm 2$

$\boxed{\frac{p}{q} = \pm 1, \pm 3, \pm 2, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}}$

b) $f(x) = 2x^3 - 2x^2 - 4x + 5$

$p = \pm 1, \pm 5$

$q = \pm 1, \pm 2$

$\boxed{\frac{p}{q} = \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}}$

Factor & Remainder Theorems

If you divide a polynomial by its factor, the remainder is zero.

If the remainder is NOT zero, the remainder is the y-value of the x-value of the factor!

☆ Use long division & the remainder theorem to check which of the POSSIBLE rational roots are zeros of the polynomial.

Ex #6 Let $f(x) = x^3 - 2x^2 - 5x + 6$.

a) List the POSSIBLE rational roots.

$\boxed{\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6}$ ← #s you can test for part b

b) Apply the Remainder Theorem to find a zero of f(x).

test $x = 2$

$$
\begin{array}{r l}
 & x^2 + 0x - 5 \\
x - 2 & \overline{\big) \, x^3 - 2x^2 - 5x + 6} \\
 & -(x^3 - 2x^2) \\
\hline
 & 0x^2 - 5x \\
 & -(0x^2 + 0x) \\
\hline
 & -5x + 6 \\
 & -(-5x + 10) \\
\hline
 & -4
\end{array}
$$

$\boxed{\text{Point @ } (2, -4)}$

test
$x = 1$

$$
\begin{array}{r}
x^2 - x - 6 \\
x - 1 \,\overline{)\, x^3 - 2x^2 - 5x + 6} \\
-(x^3 - x^2) \\
\hline
-x^2 - 5x \\
-(-x^2 + x) \\
\hline
-6x + 6 \\
-(-6x + 6) \\
\hline
0
\end{array}
$$

$\boxed{\text{x-int @ } (1,0)}$

$$6\,\overline{)\,24}\quad 4$$

c) Rewrite f(x) withits factors.

$f(x) = x^3 - 2x^2 - 5x + 6$

$f(x) = (x-1)(x^2 - x - 6)$

-6, -3, 2, -1

$f(x) = (x-1)(x-3)(x+2)$

d) Graph f(x).