9.3 Probability

Probability of an Event (equally likely outcomes)

If E is an event in a finite, nonempty sample space S of equally likely outcomes, then the probability of the event E is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the number of outcomes in } S}$$

Ex#1 Find the probability of the following events

a) Tossing a head on one toss of a fair coin. $1/2$

b) Tossing two heads in a row on two tosses of fair coins $1/4$

c) Draw a queen from a std deck of 52 cards $4/52 = 1/13$

d) Rolling a sum of 3 on a single roll of 2 fair dice $2/36 = 1/18$

e) Guessing all 6 #s in a state lottery that requires you to pick 6 numbers between 1 & 46, inclusive. $1/9{,}366{,}819$

Ex#2 Write the probability distribution for rolling 2 thru 12 on a pair of fair std dice.

2	3	4	5	6	7	8	9	10	11	12
$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

Ex#3 Find the probability of rolling a sum divisible by 3 on a single roll of two fair std dice.

$\{3, 6, 9, 12\}$ $\quad 2/36 + 5/36 + 4/36 + 1/36 = 12/36 = 1/3$

Probability Function

A function P that assigns a real number to each outcome in a sample space S subject to the conditions:

1. Probability will be between 0 & 1, inclusive.
2. The sum of all outcomes in S is 1.
3. Probability of the null set is zero.

Probability of an Event (outcomes not equally likely)

If E is any event in S, the probability of E is the sum of the individual probabilities in E.

Ex#4 Is it possible to weight a std 6-sided die so that the probability of rolling each # n is exactly $\frac{1}{n^2+1}$?

1	2	3	4	5	6
$1/2$	$1/5$	$1/10$	$1/17$	$1/26$	$1/37$

NO! sum of Probabilities $\neq 1$.

Ex#5 Sal opens a box of a dozen chocolate cremes & generously offers two to Val. Val likes vanilla cremes best, bull all the chocolates look alike on the outside. If four of the twelve cremes are vanilla, what is the probability that both of Val's picks turn out to be vanilla?

$$\frac{{}_4C_2}{{}_{12}C_2} = \frac{6}{66} = \frac{1}{11}$$

Multiplication Principle of Probability

Suppose an event A has probability p_1 and event B has probability p_2 under the assumption that A occurs. The probability of both A and B occuring is $p_1 p_2$.

Ex#6 Do ex#5 again but w/

$$P(\text{Val v.c. 1st})\,P(\text{Val v.c. 2nd}) = \frac{4}{12}\left(\frac{3}{11}\right) = \frac{12}{132} = \frac{1}{11}$$

Venn Diagrams & Tree Diagrams

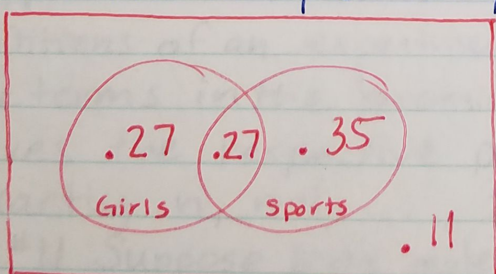
Venn: relationships among events w/i a sample space.

Tree: multiplication Principle of Counting.

Ex#7 In a high school, 54% of stds are girls & 62% of sts play sports. Half the girls play sports.

a) what % of stds that play sports are boys?

b) what is the probability that a boy does not play sports?



a) $\frac{.35}{.62} = 56.452\%$

b) $.11 = {}^{11}/_{100}$

Ex#8 Two cookie jars are identical & on a counter. Jar A has 2 CC & 2 PB cookies, while Jar B has 1 CC cookie. We select a cookie @ random. What is the probability that it is a CC cookie?

$P(CC) = P(\text{Jar A CC}) + P(\text{Jar B CC})$

$= (.125 + .125) + .5$

$P(CC) = .75$

Conditional Probability

In the previous problem, the probability of getting a cookie depended on the jar you selected. This is called conditional probability. The probability of event B given that event A occurs is denoted $P(B|A)$. Multiplication Principle of Probability leads us to
$P(B/A) = P(A \text{ and } B)/P(A)$, which is the conditional probability formula.

Ex #9 Do as in Ex#8, but given that the cookie is chocolate chip, what is the probability that it came from Jar A?

$$P(\text{Jar A}|CC) = \frac{P(\text{Jar A} \,\&\, CC)}{P(CC)} = \frac{.5(.5)}{.75} = \frac{1}{3}$$

Binomial Distributions

Ex #10 You roll a fair die 4 times. What is the prob. that you roll

(a) all 3's $\left(\frac{1}{6}\right)^4 = 0.00077$

(b) no 3's $\left(\frac{5}{6}\right)^4 = 0.48225$

(c) exactly 2 3's $\binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = 0.11574$

If $p = \frac{1}{6}$ & $q = \frac{5}{6}$ then $(p+q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

Binomial Distribution Theorem

Suppose an event occurs & consists of n independent repititions of an experiment w/ two outcomes. Then the terms in the binomial expansion of $(p+q)^n$ gives the respective probabilities of the exactly $n, n-1, \ldots, 2, 1, 0$ successes.

Ex #11 Suppose Mike makes 90% of free throws. If he shoots 20 free throws, & if his chance of making each one is independent of the others, what is

(a) P(all 20)
$.9^{20} = 0.12158$

(b) P(exactly 18)
$\binom{20}{18}.9^{18}.1^2 = 0.28578$

(c) P(at least 18) = P(ex. 18) + P(ex. 19) + P(ex. 20) = 0.6769
$\quad \binom{20}{19}.9^{19}.1^1$