7.6

Power Series (about a)

$$\sum_{n=0}^{\infty} C_n (x-a)^n \; ; \; C_n \text{ \& } a \text{ are constants}$$

this is a function of x.

Interval of Convergence

Set of all x-values for which the power series converges. You *must* check the endpoints. You will use ratio/root tests to determine the interval.

Ex#1 Determine the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n} (x+3)^n$.

$$L = \lim_{n\to\infty} \sqrt[n]{\left| \frac{(-1)^n n}{4^n} (x+3)^n \right|}$$

$$= \lim_{n\to\infty} \left| \frac{-1 \sqrt[n]{n}}{4} (x+3) \right|$$

$$L = \frac{1}{4} |x+3| < 1$$

$$|x+3| < 4$$

$x+3 < 4$
$x < 1$

$-x-3 < 4$
$-x < 7$
$x > -7$

$$-7 < x < 1$$

$x = -7$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n} (-4)^n = \sum_{n=1}^{\infty} \frac{\cancel{(-1)^n} n \cancel{(-1)^n} \cancel{(4)^n}}{\cancel{4^n}} = \sum_{n=1}^{\infty} n$

diverges by nTT

$x = 1$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n n}{\cancel{4^n}} (\cancel{4^n}) = \sum_{n=1}^{\infty} (-1)^n n$ diverges by nTT

Series converges for $\boxed{-7 < x < 1}$.

Ex#2 $\quad \sum_{n=1}^{\infty} \frac{2^n}{n} (4x-8)^n$

$$L = 2|4x-8| < 1 \qquad \boxed{\frac{15}{8} \leq x < \frac{17}{8}}$$

$x = 15/8$ converges by AST or Alternating Harmonic $\quad$ $x = 17/8$ diverges, harmonic

$\sum \frac{(-1)^n}{n}$

Ex #3 $\displaystyle\sum_{n=1}^{\infty} \frac{(x-6)^n}{n^n}$

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(x-6)^n}{n^n}\right|} = \lim_{n\to\infty} \left|\frac{x-6}{n}\right| = |x-6|(0)$$

$$= 0 < 1$$

converges for $-\infty < x < \infty$

<u>Special Case</u> $(x=a)$

$$\sum_{n=0}^{\infty} C_n (x-a)^n = \sum_{n=0}^{\infty} C_n (0)^n$$

$$= C_0 0^0 + C_1 0^1 + C_2 0^2 + \cdots$$

$$= C_0 + 0 + 0 + 0 + 0 + \cdots$$

$$= C_0$$

No matter what, a power series will converge for $x=a$ (at least this one value!)

Ex #4 $\displaystyle\sum_{n=0}^{\infty} n!\,(2x+1)^n$

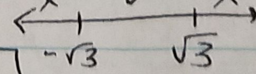
$$L = \infty > 1$$

converges for $x = -\frac{1}{2}$

Ex #5 $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{(-3)^n}$

$$L = \lim_{n\to\infty} \sqrt[n]{\left|\frac{x^{2n}}{(-3)^n}\right|} = \lim_{n\to\infty} \left|\frac{x^2}{-3}\right| = \frac{x^2}{3} < 1$$

$$x^2 < 3$$



$$\boxed{-\sqrt{3} < x < \sqrt{3}}$$

$$\sum_{n=1}^{\infty} \frac{(-\sqrt{3})^{2n}}{(-3)^n} = \sum \frac{3^n}{(-3)^n} = \sum (-1)^n \text{ diverges by nTT}$$

$$\sum_{n=1}^{\infty} \frac{(\sqrt{3})^{2n}}{(-3)^n} = \sum (-1)^n \text{ diverges by nTT}$$

EX #6 What is the sum of the series (w/i the interval of convergence) for $\sum_{n=0}^{\infty} \left(\frac{x^2-1}{2}\right)^n$?

What is the interval of convergence?

converges for $\boxed{-\sqrt{3} < x < \sqrt{3}}$

$$\sum_{n=0}^{\infty} \left(\frac{x^2-1}{2}\right)^n = \frac{1}{1-\left(\frac{x^2-1}{2}\right)}$$

$$= \frac{2}{2-(x^2-1)}$$

$$= \boxed{\frac{2}{3-x^2}}$$