2.3 Polynomial Functions of Higher Degree w/ Modeling

Polynomial Functions

Written in standard form (highest → lowest degree)

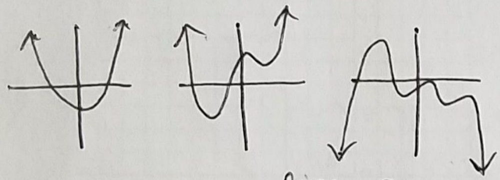
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

cubic: $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

quartic: $f(x) = a_4 x^4 + \cdots + a_0$

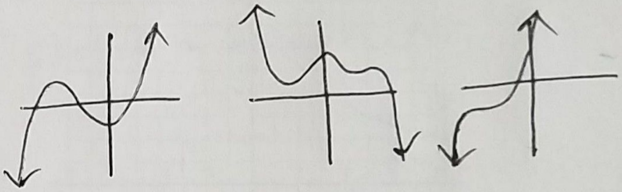
Even & Odd Degrees

Even: x^2, x^4, x^6



$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x)$

Odd: x^3, x^5, x^{11}



$\lim_{x \to \infty} f(x) = -\lim_{x \to -\infty} f(x)$

End Behavior

Leading coefficient tells you how the graph will end on the right

$f(x) = -x^2 + x - 3$

$\lim_{x \to \infty} f(x) = -\infty$

since ~~this~~ $f(x)$ is even $\lim_{x \to -\infty} f(x) = -\infty$

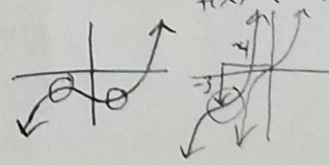
Extrema

A polynomial of degree n will have at most $n-1$ extrema.

$f(x) = x^3 - 4x^2 - 5x - 3$

This will have at most 2 extrema.

$f(x) = (x+4)^3 - 3$



Zeros/Solutions

Zeroes: 1, 2, 3, 4, 5

Solutions: $x = 1, x = 2, x = 3, \ldots$ or $x = \{1, 2, 3, 4, 5\}$

A polynomial of degree n will have exactly n solutions. This is including multiplicity and any complex solutions.

The graph will have at most n x-int.

multiplicity

Even multiplicity: bounce off of the x-axis (like a parabola)

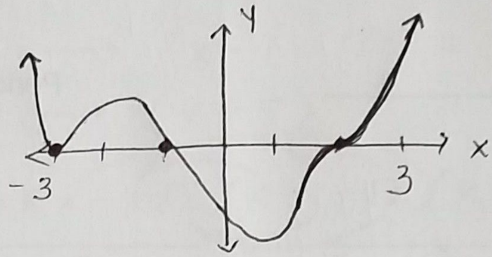
Odd multiplicity: bend through the x-axis (like a cubic)

Sketch the graph: $f(x)=(x+3)^4(x-2)^3(x+1)$ Leading term: x^8

$x=-3$ mult. 4 (bounce) $\lim_{x\to\infty} f(x)=\infty$

$x=2$ mult. 3 (bend) $\lim_{x\to-\infty} f(x)=\infty$

$x=-1$



Ex #1 Find the solutions & sketch a graph for

$f(x)=x^3-x^2-6x$.

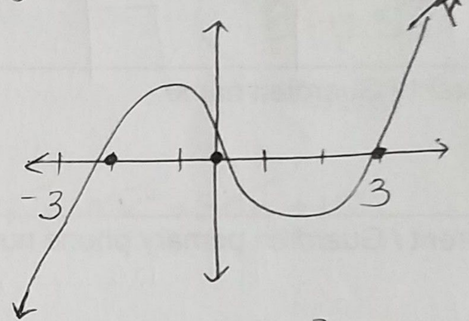
$0=x(x^2-x-6)$

$0=x(x-3)(x+2)$

$x=0$ $\lim_{x\to\infty} f(x)=\infty$

$x=3$

$x=-2$ $\lim_{x\to-\infty} f(x)=-\infty$



Ex #2 Do the same as Ex #1 for $f(x)=(x+2)^3(x-1)^2$.

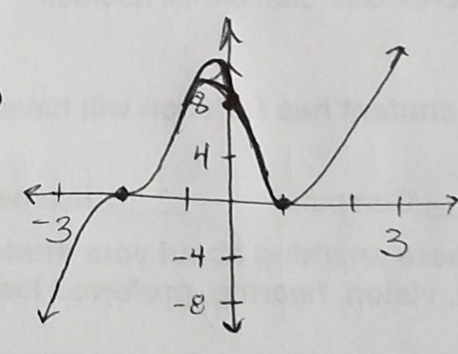
$0=(x+2)^3(x-1)^2$

$x=-2$ mult. 3 (bend)

$x=1$ mult. 2 (bounce)

$\lim_{x\to\infty} f(x)=\infty$ y-int: (0,8)

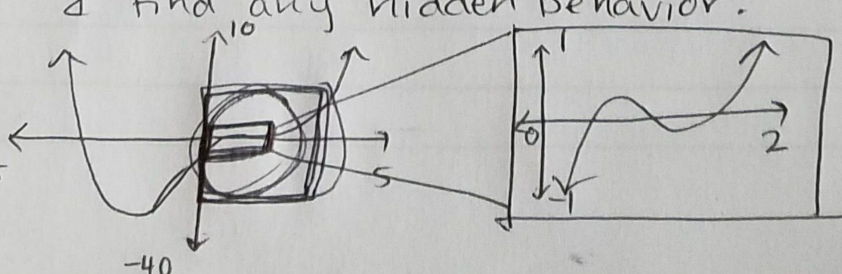
$\lim_{x\to-\infty} f(x)=-\infty$



Hidden Behavior

Zoom in/out on a graphing calculator to be sure you see everything on the graph (intercepts/extrema).

Ex #3 Graph $f(x)=x^4+0.1x^3-6.5x^2+7.9x-2.4$ & find any hidden behavior.



Write Equations Given Zeroes

If $x=a$ is a zero, then $(x-a)$ is a factor.

Ex #4 Write the simplest polynomial function with zeros $3,\ 1+\sqrt{2},\ -5$.

$x=3 \longrightarrow (x-3)$

$x=1+\sqrt{2} \longrightarrow (x-(1+\sqrt{2}))=(x-1-\sqrt{2})$

$x=1-\sqrt{2} \longrightarrow (x-(1-\sqrt{2}))=(x-1+\sqrt{2})$

$x=-5 \longrightarrow (x+5)$

$f(x)=(x-3)(x-1-\sqrt{2})(x-1+\sqrt{2})(x+5)$

$=(x-3)(x+5)(x^2-x+\cancel{x\sqrt{2}}-x+1-\cancel{\sqrt{2}}$
$\cancel{-x\sqrt{2}}+\cancel{\sqrt{2}}-2)$

$=(x-3)(x+5)(x^2-2x-1)$

$=(x^2+2x-15)(x^2-2x-1)$

$=\cancel{x^4}-\cancel{2x^3}-\cancel{x^2}+\cancel{2x^3}-\cancel{4x^2}-\cancel{2x}-\cancel{15x^2}+\cancel{30x}+15$

$$\boxed{f(x)=x^4-20x^2+28x+15}$$

DO THE HARD PART FIRST