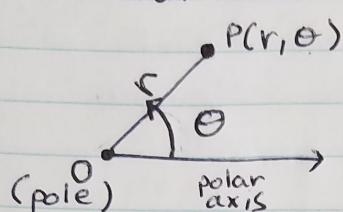


## 6.4 Polar Coordinates



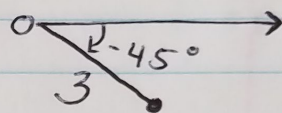
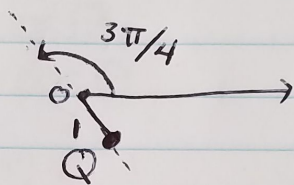
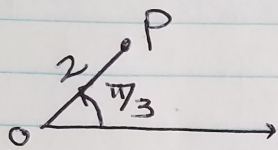
$P(r, \theta)$

$r =$  directed distance

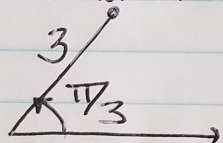
$\theta =$  directed angle

★ If  $r < 0$ , terminal side is @  $\theta + \pi$  ★

Ex #1 Plot  $P(2, \pi/3)$ ,  $Q(-1, 3\pi/4)$  &  $R(3, -45^\circ)$



Ex #2 Find at least four polar coords. equiv. to  $(3, \pi/3)$ .



$$P(3, \frac{\pi}{3})$$

$$P(-3, \frac{10\pi}{3})$$

$$P(-3, \frac{4\pi}{3})$$

$$P(3, -\frac{5\pi}{3})$$

$$P(3, \frac{13\pi}{3})$$

Add  $2n\pi, n \in \mathbb{Z}$  or make  $r$  negative & add  $(2n+1)\pi, n \in \mathbb{Z}$

### Conversions

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

$$\frac{y}{x} = \tan \theta$$

Ex #3 Convert  $(3, \frac{5\pi}{6})$  to rectangular coords.

$$x = 3 \cos \frac{5\pi}{6}$$

$$y = 3 \sin \frac{5\pi}{6}$$

$$x = 3(-\frac{\sqrt{3}}{2})$$

$$y = 3(\frac{1}{2})$$

$$x \approx -2.598$$

$$y = 1.5$$

Polar coord.  $(3, \frac{5\pi}{6}) =$  Rectangular coord.  $(-2.598, 1.5)$

E



Ex#4 Convert  $(-1, 1)$  to polar coords. & give two equivalent pts.

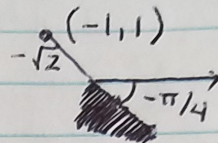
equivalent pts.

$$r = \pm\sqrt{1+1}$$

$$r = \pm\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{-1}\right)$$

$$\theta = -\pi/4$$



$$(-\sqrt{2}, -\pi/4), (\sqrt{2}, 3\pi/4)$$

Ex#5 convert  $r=4\cos\theta$  to a rectangular eqn.

$$r=4\cos\theta$$

$$r^2=4r\cos\theta$$

$$x^2+y^2=4x$$

$$x^2-4x+4+y^2=4$$

$$(x-2)^2+y^2=4$$

circle w/ radius 2 & C(2, 0).

Ex#6 convert  $r=4\sec\theta$  to a rectangular eqn.

$$r=4\sec\theta$$

$$r\cos\theta=4$$

$$x=4$$

Vertical line

Ex#7 Convert  $(x-3)^2+(y-2)^2=13$  to polar.

$$x^2-6x+9+y^2-4y+4=13$$

$$x^2-6x+y^2-4y=0$$

$$\underline{x^2+y^2-6x-4y=0}$$

$$r^2-6r\cos\theta-4r\sin\theta=0$$

$$r(r-6\cos\theta-4\sin\theta)=0$$

$$r=0 \text{ or } r=6\cos\theta+4\sin\theta$$

$$\boxed{r=6\cos\theta+4\sin\theta}$$

Ex#8 One plane is @  $(8\text{mi}, 110^\circ)$  & the second is @  $(5\text{mi}, 15^\circ)$ . How far are they from ea. other?

$$d^2=8^2+5^2-2(8)(5)\cos(110-15)$$

$$d=\sqrt{64+25-80\cos 95}$$

$$d \approx 9.797 \text{ miles apart}$$

