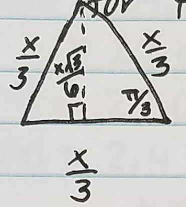
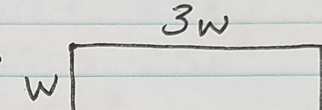
3.8

EX#1 A 30-inch string will be cut into two pieces. One piece will create an equilateral triangle, the other a rectangle whose length is 3 times its width. Where should the string be cut so that the combined area is minimized?
[How might the combined areas be maximized?]

Let x represent the length of string used for the triangle.



$h = \frac{x\sqrt{3}}{6}$



$P = 8w = 30 - x$

$w = \frac{30-x}{8}$

$A_\Delta = \frac{1}{2}\left(\frac{x}{3}\right)\left(\frac{x\sqrt{3}}{6}\right)$

$A_\Delta = \frac{x^2\sqrt{3}}{36}$

$A_\square = 3\left(\frac{30-x}{8}\right)^2$

$$A_{tot} = \frac{x^2\sqrt{3}}{36} + 3\left(\frac{30-x}{8}\right)^2$$

$$A_{tot}' = \frac{\sqrt{3}}{18}x - \frac{180-6x}{64}$$

$$x = \frac{810}{16\sqrt{3}+26} \text{ inches}$$

Since $A_{tot}' > 0$ for $x > \frac{810}{16\sqrt{3}+26}$ &
$A_{tot}' < 0$ for $x < \frac{810}{16\sqrt{3}+26}$,
then $x = \frac{810}{16\sqrt{3}+26}$ in will minimize the total area & $A\left(\frac{810}{16\sqrt{3}+26}\right) < A(0)$.

$A_{tot}(0) = 42.1875 \text{ in}^2$

$A_{tot}(30) \approx 60.622 \text{ in}^2$

Maximum area will occur when entire string is used for the triangle.

Ex#2 A toll road averaged 54,000 cars per day over the last 5 years, with a \$0.50 charge per car. A study concludes that for every \$0.05 increase, the number of cars is reduced by 500.
What toll should be charged to maximize revenue?

Let x = # of times toll is increased

$R = (\#\text{ of cars per day})(\text{cost of toll})$

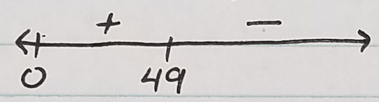
$R(x) = (54{,}000 - 500x)(0.50 + 0.05x)$

$= 27{,}000 + 2{,}700x - 250x - 25x^2$

$R(x) = 27{,}000 + 2{,}450x - 25x^2$

$R'(x) = 2{,}450 - 50x$

$x = 49$ toll increases



$R(49)$ is the maximum b/c $R'(x) > 0$ for $x < 49$ & $R'(x) < 0$ for $x > 49$.

toll $= 0.50 + 0.05(49)$

toll $= \$2.95$

Revenue is maximized when the toll is \$2.95.