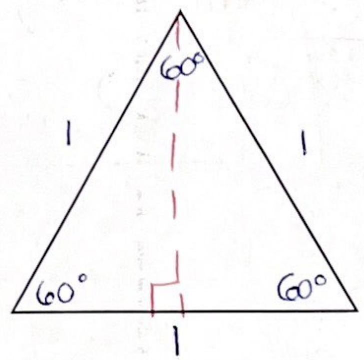
Arroyo P.6 4/22/22

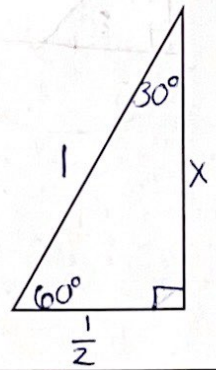
A65: The Unit Circle (Round ~~1~~ 2)

Notes: 32.2 Special Triangles & The Unit Circle

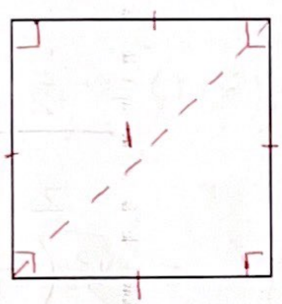
Special Triangles

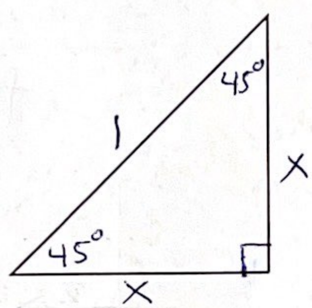
Where they come from





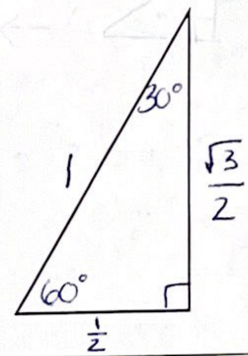
$$\left(\frac{1}{2}\right)^2 + x^2 = 1^2$$
$$\frac{1}{4} + x^2 = 1$$
$$x^2 = \frac{3}{4}$$
$$x = \sqrt{\frac{3}{4}}$$
$$x = \frac{\sqrt{3}}{2}$$

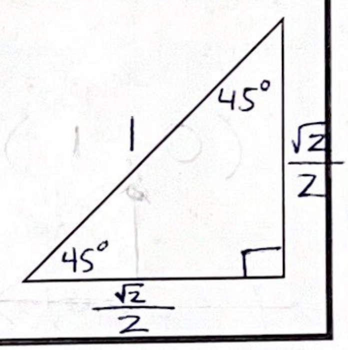




$$x^2 + x^2 = 1^2$$
$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \sqrt{\frac{1}{2}}$$
$$x = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$x = \frac{\sqrt{2}}{\sqrt{4}}$$
$$x = \frac{\sqrt{2}}{2}$$

Memorize this



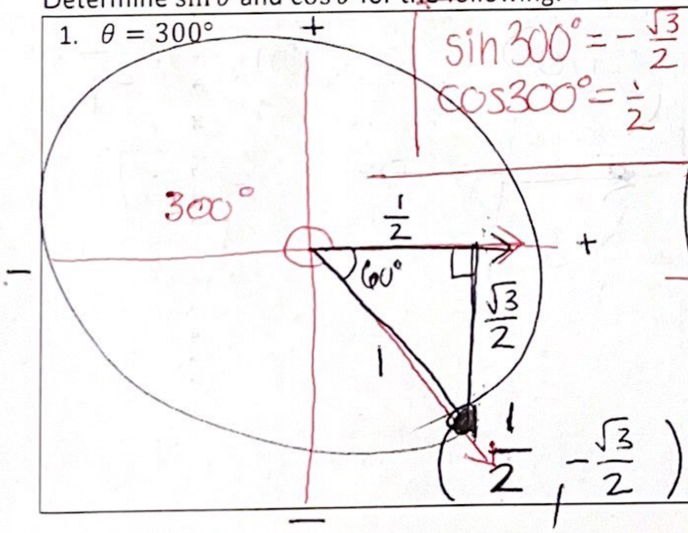


The Unit Circle and Special Angles

- Draw the angle in standard position
- Determine the reference angle α
- Draw the special triangle with α at the origin/center
- $\cos\theta = x$
- $\sin\theta = y$

Determine $\sin\theta$ and $\cos\theta$ for the following:

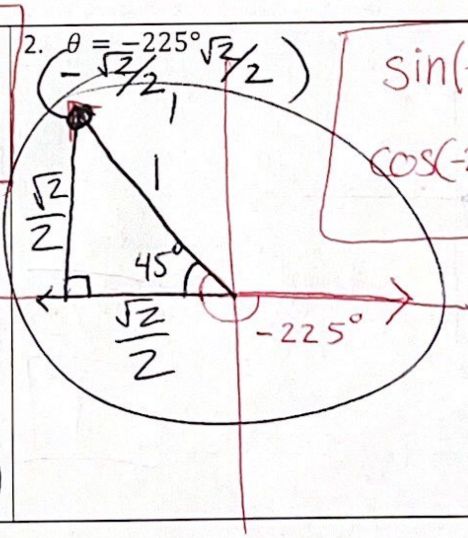
1. $\theta = 300°$



$\sin 300° = -\frac{\sqrt{3}}{2}$

$\cos 300° = \frac{1}{2}$

2. $\theta = -225°$

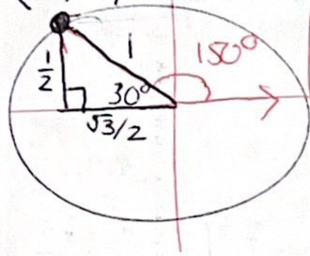


$\sin(-225°) = \frac{\sqrt{2}}{2}$

$\cos(-225°) = -\frac{\sqrt{2}}{2}$

3. $\theta = \frac{5\pi}{6}\left(\frac{180}{\pi}\right) = \frac{5(180)}{6} = 5(30) = 150^\circ$

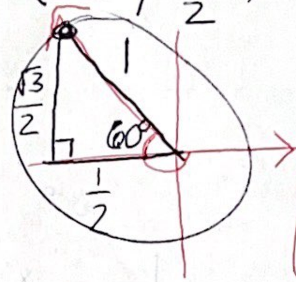
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$$\boxed{\begin{aligned}\sin\frac{5\pi}{6} &= \frac{1}{2}\\ \cos\frac{5\pi}{6} &= -\frac{\sqrt{3}}{2}\end{aligned}}$$

4. $\theta = -\frac{4\pi}{3}\left(\frac{180}{\pi}\right) = \frac{-4(180)}{3} = -4(60) = -240^\circ$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$$\boxed{\begin{aligned}\sin\left(-\frac{4\pi}{3}\right) &= \frac{\sqrt{3}}{2}\\ \cos\left(-\frac{4\pi}{3}\right) &= -\frac{1}{2}\end{aligned}}$$

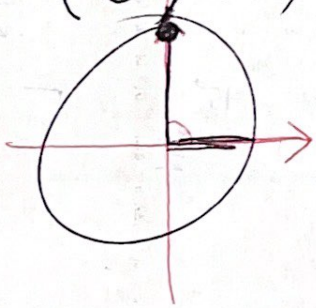
Tangent on the Unit Circle

- $\tan\theta = \frac{opposite}{adjacent} = \frac{y}{x}$

Determine $\tan\theta$ for the following:

5. $\theta = 450^\circ - 360^\circ = 90^\circ$

$(0, 1)$



$\tan(450^\circ) = \frac{1}{☹}$

$= \boxed{\text{undefined}}$

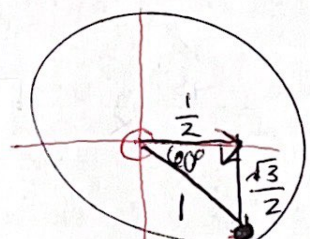
6. $\theta = 300^\circ$

$\tan(300^\circ) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= -\frac{\sqrt{3}}{2}\left(\frac{2}{1}\right)$

$= -\frac{\sqrt{3}}{1}$

$= \boxed{-\sqrt{3}}$

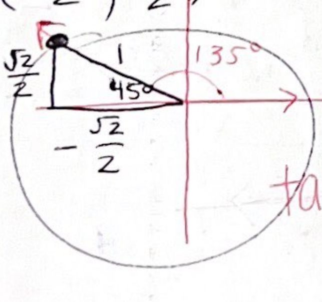


$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

7. $\theta = \frac{11\pi}{4}\left(\frac{180}{\pi}\right) = \frac{11(180)}{4} = 495^\circ$

$495^\circ - 360^\circ = 135^\circ$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

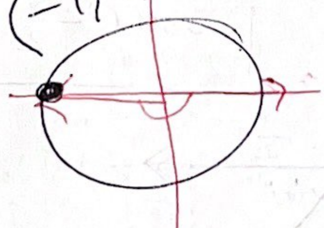


$\tan\left(\frac{11\pi}{4}\right) = \frac{\sqrt{2}/2}{-\sqrt{2}/2}$

$= \boxed{-1}$

8. $\theta = -\pi = -180^\circ$

$(-1, 0)$



$\tan(-\pi) = \frac{☺}{-1}$

$= \boxed{0}$