Notes: 33.1/33.2: Trigonometric Identities

The Quotient and Pythagorean Identities:

Remember...

On the Unit Circle:

$x = \cos\theta$

$y = \sin\theta$

$\tan\theta = \frac{y}{x}$

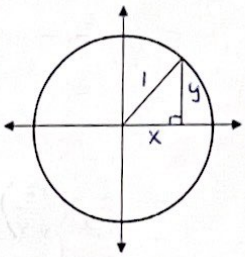
So another way to write $\tan\theta$ is....

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Remember...

Pythagorean Theorem: $a^2 + b^2 = c^2$

On the Unit Circle: $x = \cos\theta$ and $y = \sin\theta$



$x^2 + y^2 = 1^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

Quotient Identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

Use the Pythagorean Identity and then the Quotient Identity to solve the following:

1. Given that $\cos\theta = -\frac{3}{5}$ and that $\frac{\pi}{2} < \theta < \pi$, find the value of $\sin\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{3}{5}\right)^2 = 1$

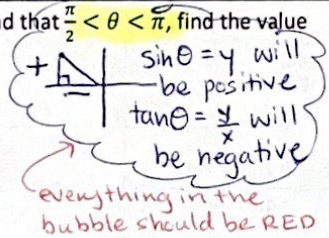
$\sin^2\theta + \frac{9}{25} = 1$

$\sin^2\theta = 1 - \frac{9}{25}$

$\sin^2\theta = \frac{25}{25} - \frac{9}{25}$

$\sin^2\theta = \frac{16}{25}$

$\sin\theta = \frac{4}{5}$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{4/5}{-3/5}$

$\tan\theta = \frac{4}{5}\left(-\frac{5}{3}\right)$

$\tan\theta = -\frac{4}{3}$

2. Given that $\cos\theta = -\frac{8}{17}$ and that $\frac{\pi}{2} < \theta < \pi$, find the value of $\sin\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{8}{17}\right)^2 = 1$

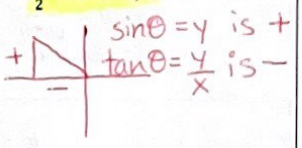
$\sin^2\theta + \frac{64}{289} = 1$

$\sin^2\theta = 1 - \frac{64}{289}$

$\sin^2\theta = \frac{289}{289} - \frac{64}{289}$

$\sin^2\theta = \frac{225}{289}$

$\sin\theta = \frac{15}{17}$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{15/17}{-8/17}$

$\tan\theta = \frac{15}{17}\left(-\frac{17}{8}\right)$

$\tan\theta = -\frac{15}{8}$

3. Given that $\sin\theta = \frac{3}{5}$ and that $\frac{\pi}{2} < \theta < \pi$, find the value of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

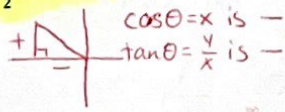
$\frac{9}{25} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{9}{25}$

$\cos^2\theta = \frac{25}{25} - \frac{9}{25}$

$\cos^2\theta = \frac{16}{25}$

$\cos\theta = -\frac{4}{5}$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}}$

$\tan\theta = \frac{3}{5}\left(-\frac{5}{4}\right)$

$\tan\theta = -\frac{3}{4}$

4. Given that $\sin\theta = -\frac{40}{41}$ and that $\pi < \theta < \frac{3\pi}{2}$, find the value of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{40}{41}\right)^2 + \cos^2\theta = 1$

$\frac{1600}{1681} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{1600}{1681}$

$\cos^2\theta = \frac{1681}{1681} - \frac{1600}{1681}$

$\cos^2\theta = \frac{81}{1681}$

$\cos\theta = -\frac{9}{41}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{-\frac{40}{41}}{-\frac{9}{41}}$

$\tan\theta = -\frac{40}{41}\left(-\frac{41}{9}\right)$

$\tan\theta = \frac{40}{9}$

↳ you can simplify earlier to $-\frac{\sqrt{3}}{2}$ ↳ $-\frac{\sqrt{2}}{2}$

5. Given that $\cos\theta = -\frac{5\sqrt{3}}{10}$ and that $\frac{\pi}{2} < \theta < \pi$, find the value of $\sin\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(-\frac{5\sqrt{3}}{10}\right)^2 = 1$

$\sin^2\theta + \frac{25(3)}{100} = 1$

$\sin^2\theta + \frac{75}{100} = 1$

$\sin^2\theta = \frac{100}{100} - \frac{75}{100}$

$\sin^2\theta = \frac{25}{100}$

$\sin\theta = \frac{5}{10}$

$\boxed{\sin\theta = \frac{1}{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

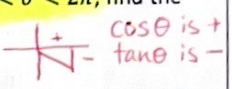
$\tan\theta = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{1}{2}\left(-\frac{2}{\sqrt{3}}\right)$

$\tan\theta = -\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

$\boxed{\tan\theta = -\frac{\sqrt{3}}{3}}$

6. Given that $\sin\theta = -\frac{4\sqrt{2}}{8}$ and that $\frac{3\pi}{2} < \theta < 2\pi$, find the value of $\cos\theta$ and $\tan\theta$.



$\sin^2\theta + \cos^2\theta = 1$

$\left(-\frac{\sqrt{2}}{2}\right)^2 + \cos^2\theta = 1$

$\frac{2}{4} + \cos^2\theta = 1$

$\frac{1}{2} + \cos^2\theta = 1$

$\cos^2\theta = \frac{2}{2} - \frac{1}{2}$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$\boxed{\cos\theta = \frac{\sqrt{2}}{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{-\sqrt{2}/2}{\sqrt{2}/2}$

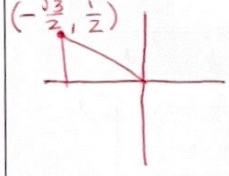
$\boxed{\tan\theta = -1}$

The Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$ "cosecant"	$\cos\theta = \frac{1}{\sec\theta}$ "secant"	$\tan\theta = \frac{1}{\cot\theta}$ "cotangent"
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y} = \frac{hypotenuse}{opposite}$	$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x} = \frac{hypotenuse}{adjacent}$	$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{adjacent}{opposite}$

Find the exact value of the following trigonometric expressions:

7. $\csc(-210°) = \csc(-210° + 360°) = \csc(150°)$

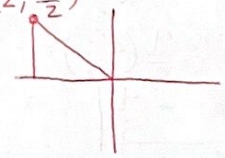


$= \frac{1}{\frac{1}{2}}$

$= 1\left(\frac{2}{1}\right)$

$= \boxed{2}$

8. $\cot\left(\frac{5\pi}{6}\right)$

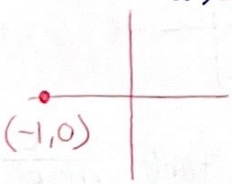


$= \frac{-\sqrt{3}/2}{1/2}$

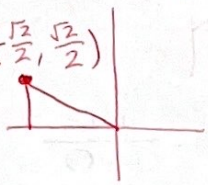
$= -\frac{\sqrt{3}}{2}\left(\frac{2}{1}\right)$

$= \boxed{-\sqrt{3}}$

9. $\sec(-\pi) = \sec(\pi) = \frac{1}{-1} = \boxed{-1}$

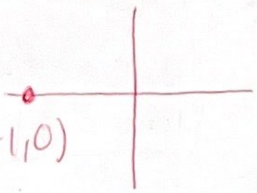


10. $\cot(135°)$



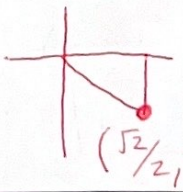
$= \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$

11. $\csc(\pi)$



$= \frac{1}{0} = \boxed{\text{undefined}}$

12. $\sec(315°)$



$= \frac{1}{\sqrt{2}/2} = 1\left(\frac{2}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$