7.2 Matrix Algebra

Vocabulary

Matrix: rectangular array of numbers $A = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}$

$m \times n$ matrix: a matrix w/ m rows & n columns

row: horizontal $\longleftrightarrow$

column: vertical $\updownarrow$

Element $[a_{ij}]$: an entry in the i^{th} row & j^{th} column

Order: $m \times n$

Square matrix: a matrix where $i = j$ $(m = n)$.

Equal matrices: have same order & corresponding elements

Scalar: a real number you might mult. a matrix by.

Zero Matrix: a matrix w/ only zeros in it
AKA additive identity.

Additive inverse matrix: a matrix B that has exact opposite entries of A so that when added get $[0]$

Identity matrix: a sq matrix w/ ones accross the diagonal (upper left to lower right) & zeros elsewhere AKA multiplicative identity $[I]$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ex #1 Determine the order of the matrices.

a) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$ $\boxed{2 \times 3}$

$B = \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 2 & -1 \\ 3 & 2 \end{bmatrix}$ $\boxed{4 \times 2}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\boxed{3 \times 3}$

Matrix Addition/Subtraction - must have same order
add/sub corresponding entries

Ex #2: $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 2 \\ -2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 5 \\ 0 & 4 & 7 \end{bmatrix}$

Scalar Multiplication - distribute the scalar

Ex #3: $4\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -8 & 12 \\ 8 & 0 & 16 \end{bmatrix}$

Matrix Multiplication - # of columns in A = # of rows in B where you mult. in the order AB. Mult entries of row 1 of A to corresponding entries in column 1 of B, then add together & that's what goes in AB.

$$\begin{array}{cc} A & B \\ \downarrow & \downarrow \end{array}$$

Ex#4: $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1-8+21 & 2-10+24 & 3-12+ \\ 2+0+28 & 4+0+32 & 6+0+36 \end{bmatrix}$

$$AB = \begin{bmatrix} 14 & 16 & 18 \\ 30 & 36 & 42 \end{bmatrix}$$

Ex#5: BA is not possible

Ex#6:

$$A = \begin{array}{l} \\ \text{Roses} \\ \text{Carnations} \\ \text{Lilies} \end{array} \begin{array}{c} \begin{array}{ccc} \text{I} & \text{II} & \text{III} \end{array} \\ \begin{bmatrix} 5 & 8 & 7 \\ 6 & 6 & 7 \\ 4 & 3 & 3 \end{bmatrix} \end{array}$$

$$B = \begin{array}{l} \text{Roses} \\ \text{carnations} \\ \text{Lilies} \end{array} \begin{bmatrix} 1.50 & 1.35 \\ 0.95 & 1.00 \\ 1.30 & 1.35 \end{bmatrix}$$

$$\begin{array}{cc} & \text{W1} \quad \text{W2} \end{array}$$

A transpose $\rightarrow$ $A^T = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{ccc} R & C & L \end{array} \\ \begin{bmatrix} 5 & 6 & 4 \\ 8 & 6 & 3 \\ 7 & 7 & 3 \end{bmatrix} \end{array}$ ★need A^T to match flower w/ corresponding price★

$$A^T B = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{ccc} R & C & L \end{array} \\ \begin{bmatrix} 5 & 6 & 4 \\ 8 & 6 & 3 \\ 7 & 7 & 3 \end{bmatrix} \end{array} \begin{array}{c} \\ R \\ C \\ L \end{array} \begin{array}{c} \begin{array}{cc} \text{W1} & \text{W2} \end{array} \\ \begin{bmatrix} 1.50 & 1.35 \\ 0.95 & 1.00 \\ 1.30 & 1.35 \end{bmatrix} \end{array} = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{cc} \text{W1} & \text{W2} \end{array} \\ \begin{bmatrix} 18.40 & 18.15 \\ 21.60 & 20.85 \\ 21.05 & 20.50 \end{bmatrix} \end{array}$$

Identity and Inverse Matrices - only sq matrices have inverses, but not all sq matrices have an inverse.

B is the inverse of A iff $AB = BA = I$; $B = A^{-1}$

If A has an inverse → nonsingular, if it has no inverse → singular

Ex#7: Prove A & B are inverses of ea. other.

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Determinant - a test/number that tells us if a matrix has an inverse. The $n \times n$ matrix has an inverse iff $\det A = |A| \neq 0$

Ex#8: $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ $\det A = 2$ ∴ A has an inverse

Ex#9: $B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$ $|B| = -10$ ∴ B has an inverse

$$\det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$