3.6 Mathematics of Finance

Interest Compounded k Times per Year

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

A = amt in the acct after t yrs
P = principal (initial investment)
r = fixed annual rate (as a decimal)
k = number of compounds per year
t = number of years

Continually Compounded Interest

$$A = Pe^{rt}$$

A = amt in the account after t yrs
P = principal
e = Euler's # ≈ 2.71828
r = annual rate (decimal)
t = yrs

Annual Percentage Yield (~~AYP~~ APY)

~~AYP~~ APY tells you the rate you would get from an annual compound even though the actual account might compound differently.

Annuities

Have a periodic payment, not just an initial investment.

$$FV = R\,\frac{(1+i)^n - 1}{i}$$

$$PV = R\,\frac{1-(1+i)^{-n}}{i}$$

FV = Future value (eg. retirement, college fund)
PV = Present value (eg. loan, mortgage payment)
R = amount put into the account periodically
$i = \dfrac{\text{rate as a decimal (yearly)}}{\text{number of compounds per yr}}$
n = (number of yrs)(number of compounds per yr)

Ex #1 You put \$5,000 into an account with an annual interest rate of 1.6%. Determine the value of the account after 20 yrs if (a) compounded annually, (b) compounded monthly & (c) compounded continually.

a) $A = 5{,}000\left(1 + \frac{0.016}{1}\right)^{20}$
$A = \$6{,}868.22$

b) $A = 5{,}000\left(1 + \frac{0.016}{12}\right)^{20(12)}$
$A = \$6{,}884.17$

c) $A = 5{,}000e^{0.016(20)}$
$A = \$6{,}885.64$

Ex #2 What is better: 8.75% compounded quarterly or 8.7% compounded monthly?
Assume initial investment is \$100.

r_1 = APY for 8.75% quarterly
r_2 = " " 8.7% monthly.

$$\cancel{100}\left(1+\frac{0.0875}{4}\right)^{4(1)} = \cancel{100}\left(1+\frac{r_1}{1}\right)^{1(1)}$$

$$\left(1+\frac{0.0875}{4}\right)^{4} = 1+r_1$$

$$r_1 = \left(1+\frac{0.0875}{4}\right)^4 - 1$$

$$r_1 \approx 0.0904131928$$

(8.75% quarterly $\rightarrow \sim$9.04% annually)

$$100\left(1+\frac{0.087}{12}\right)^{12(1)} = 100\left(1+\frac{r_2}{1}\right)^{1(1)}$$

$$\left(1+\frac{0.087}{12}\right)^{12(1)} = 1+r_2$$

$$r_2 \approx 0.09055$$

(8.7% monthly $\rightarrow \sim$9.05% yearly)

Rate 2 is better

Ex #3 You buy a car for \$18,500. What would the monthly payment be if you had a down payment of \$2000 & a 4 yr loan @ a rate of 2.9%.

$PV = 16{,}500 \qquad i = \frac{0.029}{12} \qquad n = 4(12)$

$$PV = R\,\frac{1-(1+i)^{-n}}{i}$$

$$\left(\frac{0.029}{12}\right) 16{,}500 = R\,\frac{1-\left(1+\frac{0.029}{12}\right)^{-48}}{\cancel{\frac{0.029}{12}}}\left(\cancel{\frac{0.029}{12}}\right)$$

$$\frac{16{,}500\left(\frac{0.029}{12}\right)}{1-\left(1+\frac{0.029}{12}\right)^{-48}} = \frac{R\left(\cancel{1-\left(1+\frac{0.029}{12}\right)^{-48}}\right)}{\cancel{1-\left(1+\frac{0.029}{12}\right)^{-48}}}$$

$$R = \frac{16{,}500\left(\frac{0.029}{12}\right)}{1-\left(1+\frac{0.029}{12}\right)^{-48}}$$

$$R \approx \$364.49$$