

## 22.3/22.4 Logarithmic Properties

| Exponent Property           | → | Logarithmic property   | Example   |
|-----------------------------|---|--|---|
| $b^n \cdot b^m = b^{n+m}$   | → | $\log(nm) = \log n + \log m$<br>multiplication → addition                  | $\log 36 = \log(6 \cdot 6) = \log 6 + \log 6$<br>$\log 36 = \log(4 \cdot 9) = \log 4 + \log 9$<br>$\log 36 = \log(12 \cdot 3) = \log 12 + \log 3$ |
| $\frac{b^n}{b^m} = b^{n-m}$ | → | $\log\left(\frac{n}{m}\right) = \log n - \log m$<br>division → subtraction | $\log\left(\frac{7}{4}\right) = \log 7 - \log 4$<br>$\log\left(\frac{2}{3}\right) = \log 2 - \log 3$  |
| $(b^n)^m = b^{n \cdot m}$   | → | $\log(m^n) = n \cdot \log m$<br>power → multiplication                     | $\log 36 = \log(6^2) = 2 \log 6$<br>$\log 8 = \log(2^3) = 3 \log 2$   |

Ex #1 Use Logarithmic Properties to rewrite the following:

a)  $\log\left(\frac{8}{3}\right) = \log 8 - \log 3$

b)  $\log 24 = \log 6 + \log 4$  OR  $\log 8 + \log 3$  OR ...

c)  $\log 64 = \log 8 + \log 8 = 2 \log 8$

d)  $\log 27 = \log 3 + \log 9$

e)  $\log\left(\frac{4}{9}\right) = \log 4 - \log 9$

f)  $\log 144 = 2 \log 12$

g)  $\log 81 = 2 \log 9$

Expand a Logarithm - Use log properties to rewrite a expression into the longest you can make it.

Ex #2 Expand the logs:

a)  $\log\left(\frac{8m}{9n}\right) = \log(8m) - \log(9n)$   
 $= \log 8 + \log m - (\log 9 + \log n)$   
 $= \log 8 + \log m - \log 9 - \log n$

b)  $\log(5 \times y^4) = \log 5 + \log x + \log(y^4)$   
 $= \log 5 + \log x + 4 \log y$

c)  $\log\left(\frac{x}{y^3}\right) = \log x - 3 \log y$

d)  $\log(bc^3d^2) = \log b + 3 \log c + 2 \log d$

Condense a Logarithm - Use log properties to rewrite a long expression into a single logarithm.

Ex #3 Condense the log.

$$a) \log x - \log 7 = \boxed{\log\left(\frac{x}{7}\right)}$$

$$b) 2\log x + \log y = \log x^2 + \log y \\ = \boxed{\log(x^2 y)}$$

★ when condensing, deal with any coefficients first!

$$c) 3\log 2 + \log 2 - \log 4 = \log 2^3 + \log 2 - \log 4 \\ = \log 8 + \log 2 - \log 4 \\ = \log 16 - \log 4 \\ = \log \frac{16}{4} \\ = \boxed{\log 4}$$

$$d) \log 7 + \log x - \log 3 - \log y = \log 7x - \log 3 - \log y \\ = \log 7x - (\log 3 + \log y) \\ = \log 7x - \log 3y \\ = \boxed{\log\left(\frac{7x}{3y}\right)}$$

Ex #4 Condense, then evaluate:

$$a) \log 2 + \log 5 = \log 10 = \boxed{1}$$

$$b) \log 5000 - \log 5 = \log 1000 = \boxed{3}$$

$$c) 2\log 5 + \log 4 = \log 25 + \log 4 \\ = \log 100 \\ = \boxed{2}$$

A44: Log Props Wks