A45: Pg 369 #1-37

## 23.1/23.2 Logarithms in Other Bases & Their Properties

### Logarithms in Other Bases

$$\log_b x = y \longleftrightarrow b^y = x$$

where
$x > 0$
$b > 0$
$b \neq 1$

Note: The common log, written $\log x$, is actually $\log_{10} x$.

Ex#1 Evaluate

a) $\log_5 25$ $\quad 5^? = 25$ $\quad \boxed{\log_5 25 = 2}$

b) $\log_2 8$ $\quad 2^? = 8$ $\quad \boxed{\log_2 8 = 3}$

c) $\log_4 \left(\frac{1}{64}\right)$ $\quad 4^? = \frac{1}{64}$ $\quad \boxed{\log_4 \left(\frac{1}{64}\right) = -3}$

d) $\log_{\frac{1}{2}} \left(\frac{1}{16}\right)$ $\quad \left(\frac{1}{2}\right)^? = \frac{1}{16}$ $\quad \boxed{\log_{\frac{1}{2}} \left(\frac{1}{16}\right) = 4}$

### Inverses

Logs and exponents are inverses, $f(x) = \log_b x$ has the inverse $g(x) = b^x$. When you compose them, they "undo" eachother. In other words, $f(g(x)) = g(f(x)) = x$.

Ex#2 Write the inverse, $f^{-1}(x)$, for the following:

a) $f(x) = \log_4 x$ $\quad \boxed{f^{-1}(x) = 4^x}$

b) $f(x) = 5^x$ $\quad \boxed{f^{-1}(x) = \log_5 x}$

c) $f(x) = \log_e x = \ln x$ $\quad \boxed{f^{-1}(x) = e^x}$

d) $f(x) = 3^x$ $\quad \boxed{f^{-1}(x) = \log_3 x}$

Note: The natural log, written $\ln x$, is actually $\log_e x$.

e) $f(x) = 3x - 8$

$y = 3x - 8$

$x = 3y - 8$

$x + 8 = 3y$

$\frac{x+8}{3} = y$

$\boxed{f^{-1}(x) = \frac{x+8}{3}}$

f) $f(x) = \frac{1}{2}x + 5$

$y = \frac{1}{2}x + 5$

$x = \frac{1}{2}y + 5$

$x - 5 = \frac{1}{2}y$

$2x - 10 = y$

$\boxed{f^{-1}(x) = 2x - 10}$

# Inverse Properties of Logarithms

| Inverse Property | Explanation | Example |
|---|---|---|
| $\log_b \boldsymbol{b}^x = x$ | $b^x = \boldsymbol{b}^x$ | $\cancel{\log_3}(\cancel{3}^x) = x$<br>$\cancel{\log_{\frac{1}{2}}}(\cancel{\tfrac{1}{2}}^x) = x$<br>$\cancel{\log_{10}}\cancel{10}^x = x$ |
| $\boldsymbol{b}^{\log_b x} = x$ | The log says "b to the what power gives me $x$"="the power needed on b to get $x$," so $\boldsymbol{b}^{\text{power needed on b to get } x} = x$ | $\cancel{6}^{\cancel{\log_6} x} = x$<br>$\cancel{7}^{\cancel{\log_7} x} = x$<br>$\cancel{e}^{\cancel{\ln_e} x} = x$ |

Ex#3 Simplify

a) $\cancel{\log_9 9}^x = \boxed{x}$

b) $\cancel{15}^{\cancel{\log_{15}} x} = \boxed{x}$

c) $\cancel{\ln e}^x = \boxed{x}$

d) $\cancel{8}^{\cancel{\log_8} x} = \boxed{x}$

## Log Properties

The product, quotient, & power properties still work for logs of different bases.

Ex#4 Expand

a) $\log_7\left(\frac{x}{y^3}\right) = \log_7 x - \log_7 y^3 = \boxed{\log_7 x - 3\log_7 y}$

b) $\log_4 x^2 y = \log_4 x^2 + \log_4 y = \boxed{2\log_4 x + \log_4 y}$

Ex#5 Condense

a) $\log_5 17 + 2\log_5 3 = \log_5 17 + \log_5 3^2 = \boxed{\log_5(17\cdot 3^2)}$

b) $3\ln x - \frac{1}{2}\ln y = \ln x^3 - \ln y^{1/2} = \boxed{\ln\left(\frac{x^3}{y^{1/2}}\right)}$

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