5.5 Logistic Curves

Logistic Equation: $P(t) = \dfrac{m}{1+Ce^{-rmt}}$

where m, C, and r are constants

Differential Equation to a Logistic: $\dfrac{dP}{dt} = r(m-P)P$

where m and r are constants

if you don't trust me...

$$P(t) = \frac{m}{1+Ce^{-rmt}}$$

get rid of neg. expo. $$P(t) = \frac{m}{1+Ce^{-rmt}}\left(\frac{e^{rmt}}{e^{rmt}}\right)$$

rewrite $$P(t) = \frac{me^{rmt}}{e^{rmt}+C} = me^{rmt}(e^{rmt}+C)^{-1}$$

product rule $$\frac{dP}{dt} = (me^{rmt}rm)(e^{rmt}+C)^{-1} + (me^{rmt})\left(-(e^{rmt}+C)^{-2}(e^{rmt}rm)\right)$$

rewrite $$\frac{dP}{dt} = \frac{me^{rmt}}{e^{rmt}+C}rm - r\left(\frac{me^{rmt}}{e^{rmt}+C}\right)^2$$

substitute $$\frac{dP}{dt} = Prm - rP^2$$

factor $$dP/dt = rP(m-P)$$

rewrite $$dP/dt = r(m-P)P$$

<u>What you need to know</u>

C = constant determined by initial condition

r = growth constant

m = carrying capacity

as $t \to \infty$, $P(t) \to m$

P is changing fastest when $P = \frac{m}{2}$

dP/dt is maximized when $P = \frac{m}{2}$

Not enough room for example...

(#1190) A gorilla preserve can have no more than 250 gorillas on it. In 1970 ($t=0$) there are 28 gorillas on the preserve. If the gorilla population on the preserve is modeled by $P(t) = P$, ~~and~~ t = years since 1970, and $dP/dt = 0.0004(250-P)P$, find $P(t)$, find when $P = 250$, and what is P when dP/dt is maximized?

know: $P(t) = \dfrac{m}{1+Ce^{-rmt}}$; $\dfrac{dP}{dt} = r(m-P)P$; $m = 250$;

$r = 0.0004$

want: $P(t)$ (with numbers); t when $P = 250$;

P when dP/dt maximized

$$28 = \frac{250}{1+Ce^{-0.0004(250)(0)}}$$

$$28 = \frac{250}{1+C}$$

$$1+C = \frac{250}{28}$$

$$C = \frac{250}{28} - 1$$

$$C = \frac{111}{14}$$

$$\boxed{P(t) = \frac{250}{1+\frac{111}{14}e^{-0.0004(250)t}}}$$

$$250 = \frac{250}{1+\frac{111}{14}e^{-0.0004(250)t}}$$ has no solution!!

but we can use

$$249.001 = \frac{250}{1+\frac{111}{14}e^{-0.0004(250)t}}$$

since 249.001 rounded up to the next whole gorilla is 250 to get $\approx$ $\boxed{\text{75 or 76 years}}$ for population to reach its maximum.

dP/dt (P(t) is changing fastest) is maximized when $P = \frac{m}{2} = \frac{250}{2} = 125$

rate at which gorilla pop. is changing is maximized when the gorilla population is $\boxed{\text{125 gorillas}}$.