

5.5 Logistic Curves

Logistic Equation: $P(t) = \frac{m}{1 + Ce^{-rmt}}$

where $m, C,$ and r are constants

Differential Equation to a Logistic: $\frac{dP}{dt} = r(m - P)P$

where M and r are constants

if you don't trust me...

$$P(t) = \frac{m}{1 + Ce^{-rmt}}$$

get rid of
neg. expo.

$$P(t) = \frac{m}{1 + Ce^{-rmt}} \left(\frac{e^{rmt}}{e^{rmt}} \right)$$

rewrite

$$P(t) = \frac{Me^{rmt}}{e^{rmt} + C} = Me^{rmt} (e^{rmt} + C)^{-1}$$

product
rule

$$\frac{dP}{dt} = (Me^{rmt} r m) (e^{rmt} + C)^{-1} + (Me^{rmt}) \left(-(e^{rmt} + C)^{-2} (e^{rmt} r m) \right)$$

rewrite

$$\frac{dP}{dt} = \frac{Me^{rmt}}{e^{rmt} + C} r m - r \left(\frac{Me^{rmt}}{e^{rmt} + C} \right)^2$$

substitute

$$\frac{dP}{dt} = P r m - r P^2$$

factor

$$\frac{dP}{dt} = r P (m - P)$$

rewrite

$$\frac{dP}{dt} = r(m - P)P$$

What you need to know

C = constant determined by initial condition

r = growth constant

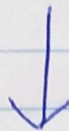
M = carrying capacity

as $t \rightarrow \infty, P(t) \rightarrow M$

P is changing fastest when $P = \frac{m}{2}$

$\frac{dP}{dt}$ is maximized when $P = \frac{m}{2}$

not enough room for example ...



(# 1190) A gorilla preserve can have no more than 250 gorillas on it. In 1970 ($t=0$) there are 28 gorillas on the preserve. If the gorilla population on the preserve is modeled by $P(t) = \frac{m}{1 + Ce^{-rmt}}$, and $t =$ years since 1970, and $\frac{dP}{dt} = 0.0004(250 - P)P$, find $P(t)$, find when $P = 250$, and what is P when $\frac{dP}{dt}$ is maximized?

know: $P(t) = \frac{m}{1 + Ce^{-rmt}}$; $\frac{dP}{dt} = r(m - P)P$; $m = 250$;

$r = 0.0004$

want: $P(t)$ (with numbers); t when $P = 250$;
 P when $\frac{dP}{dt}$ maximized

$$28 = \frac{250}{1 + Ce^{-0.0004(250)(0)}}$$

$$28 = \frac{250}{1 + C}$$

$$1 + C = \frac{250}{28}$$

$$C = \frac{250}{28} - 1$$

$$C = \frac{111}{14}$$

$$P(t) = \frac{250}{1 + \frac{111}{14}e^{-0.0004(250)t}}$$

$$250 = \frac{250}{1 + \frac{111}{14}e^{-0.0004(250)t}} \text{ has no solution } \wedge$$

but we can use

$$249.001 = \frac{250}{1 + \frac{111}{14}e^{-0.0004(250)t}}$$

since 249.001 rounded up to the next whole gorilla is 250 to get \approx 75 or 76 years for population to reach its maximum.

$\frac{dP}{dt}$ is ^{$P(t)$ is (changing fastest)} maximized when $P = \frac{m}{2} = \frac{250}{2} = 125$
 rate at which gorilla pop. is changing is maximized when the gorilla population is 125 gorillas.