2.1 Linear & Quadratic Functions & Modeling

Polynomial Function

Nonnegative integer exponents & $a_k \in \mathbb{R}$ & $a_n \neq 0$.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

Degree: n (highest exponent)

Leading Coeff: a_n (coeff of the highest exponent)

Set of coeff: $\{a_n, a_{n-1}, \ldots, a_2, a_1, a_0\}$

constant: a_0 (AKA y-intercept)

Linear Function

$y = mx + b$ slope-intercept form

m = average rate of change = AROC = $\frac{\Delta y}{\Delta x}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

$y - y_1 = m(x - x_1)$ point-slope form

$Ax + By = C$ standard form

Quadratic Functions

standard form: $y = ax^2 + bx + c$

AOS/axis: $x = \frac{-b}{2a}$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

y-int: $(0, c)$

x-int: $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$

vertex form: $y = a(x-h)^2 + k$

AOS/axis: $x = h$

vertex: (h, k)

★ y-int: use $k = c - ah^2$ (c: std form)

x-int: set $y = 0$ & solve

vertical free-fall: $g \approx 32\ ft/s^2 \approx 9.8\ m/s^2$

height: $s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

vertical velocity: $v(t) = -gt + v_0$ (speed & direction)

v_0: initial velocity

s_0: initial height

EX#1 Complete the square for the function $y = 5x^2 - 25x + 12$.

$y = 5x^2 - 25x + 12$

$y - 12 = 5x^2 - 25x$

$y - 12 = 5(x^2 - 5x)$

$y - 12 + 5\left(\frac{-5}{2}\right)^2 = 5\left(x^2 - 5x + \left(\frac{-5}{2}\right)^2\right)$

$y - 12 + 5\left(\frac{25}{4}\right) = 5\left(x - \frac{5}{2}\right)^2$

$y - 12 + \frac{125}{4} = 5(x - 5/2)^2$

$y - \frac{48}{4} + \frac{125}{4} = 5(x - 5/2)^2$

$y + \frac{77}{4} = 5(x - 5/2)^2$

$y = 5(x - 5/2)^2 - \frac{77}{4}$

EX#2 Write an equ for the graph that contains the vertex $(-2, -5)$ & point $(-4, -27)$.

$y = a(x-h)^2 + k$

$-27 = a(-4+2)^2 - 5$ → $-22 = 4a$

$a = -\frac{22}{4} = -\frac{11}{2}$

$\boxed{y = -\frac{11}{2}(x+2)^2 - 5}$