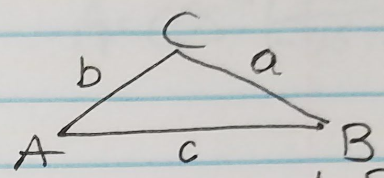
5.6 Law of Cosines

Use the Law of Cosines whenever possible, even when you think using the Law of Sines is a better idea.

If you're given $\triangle ABC$, then the following is true.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

use this w/ SSS or SAS

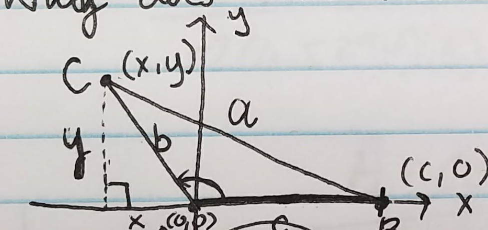
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Why does this work??

$x = b\cos A$ $\qquad$ $y = b\sin A$

$$\cos A = \frac{x}{b} \qquad \sin A = \frac{y}{b}$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$a = \sqrt{(x-c)^2 + (y-0)^2}$$

$$= \sqrt{(b\cos A - c)^2 + (b\sin A)^2}$$

$(b\cos A - c)(b\cos A - c)$

$b^2\cos^2 A - bc\cos A - bc\cos A + c^2$

$$= \sqrt{b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A}$$

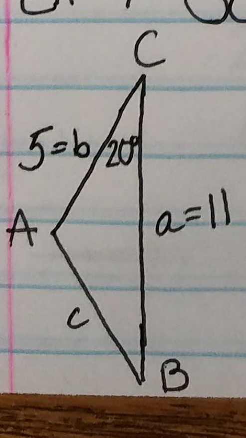
$$= \sqrt{b^2\cos^2 A + b^2\sin^2 A - 2bc\cos A + c^2}$$

$$= \sqrt{b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2}$$

$$a = \sqrt{b^2 - 2bc\cos A + c^2}$$

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

Ex #1 Solve $\triangle ABC$ given $a = 11$, $b = 5$ & $C = 20°$.



$$c^2 = b^2 + a^2 - 2ab\cos C$$

$$c^2 = 5^2 + 11^2 - 2(5)(11)\cos 20°$$

$$c = \sqrt{25 + 121 - 110\cos 20°}$$

$$c = \sqrt{146 - 110\cos 20°}$$

$$c \approx 6.5294$$

$11^2 = 6.5294^2 + 5^2 - 2(6.5294)(5)\cos A$

$121 = 42.6339 + 25 - 65.294\cos A$

$53.3661 = -65.294\cos A \qquad A = \cos^{-1}\left(\frac{53.3661}{-65.294}\right)$

$A = 144.8174^\circ$

~~∠A 180 - A - B ∠1~~

$B = 180 - A - C \approx 15.1831^\circ$

$11^2 = 6.5294^2 + 5^2 - 2(6.5294)(5)\cos A$

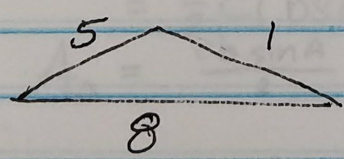
$11^2 - 6.5294^2 - 5^2 = -2(6.5294)(5)\cos A$

$$\frac{11^2 - 6.5294^2 - 5^2}{-2(6.5294)(5)} = \cos A$$

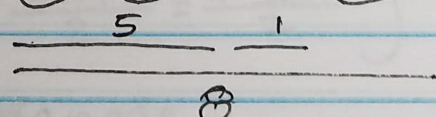
$$A = \cos^{-1}\left(\frac{11^2 - 6.5294^2 - 5^2}{-2(6.5294)(5)}\right)$$

~~∠A =~~	∠B =	∠C =
a =	b =	c =

Ex #2 Solve $\triangle ABC$ given $a = 1, b = 5, c = 8$.

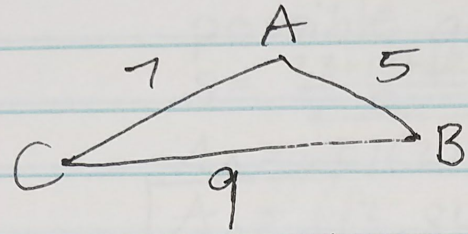


NOT POSSIBLE



2 short sides must add up to larger than the 3rd side.

Ex #3 Solve $\triangle ABC$ given $a=9, b=7, c=5$.



$$9^2 = 7^2 + 5^2 - 2(7)(5)\cos A$$

$$A = \cos^{-1}\left(\frac{9^2-7^2-5^2}{-2(7)(5)}\right)$$

$$A \approx 95.7391^\circ$$

$$7^2 = 9^2+5^2-2(9)(5)\cos B$$

$$\frac{7^2-9^2-5^2}{-2(9)(5)} = \frac{-2(9)(5)\cos B}{-2(9)(5)}$$

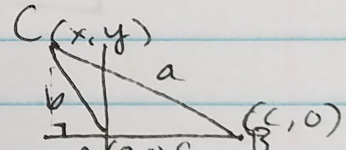
$$B = \cos^{-1}\left(\frac{7^2-9^2-5^2}{-2(9)(5)}\right)$$

$$B \approx 50.7035^\circ$$

$$C = \cos^{-1}\left(\frac{5^2-9^2-7^2}{-2(9)(7)}\right) \approx 33.5573^\circ$$

$\angle A = 95.739^\circ$	$\angle B = 50.704^\circ$	$\angle C = 33.557^\circ$
$a = 9$	$b = 7$	$c = 5$

Area & Heron's Formula



$$A_\Delta = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}cy$$
$$= \frac{1}{2}c(b\sin A)$$

$$A_\Delta = \frac{cb\sin A}{2} = \frac{ab\sin C}{2} = \frac{ac\sin B}{2}$$ } use when given SAS

Given SSS triangle

semiperimeter $= s = \dfrac{a+b+c}{2}$

$$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$ } use when given SSS

Ex#4 Find the area of $\triangle ABC$ if possible given $a=23, b=19, c=12$.

$S = \frac{23+19+12}{2} = 27$ $\qquad 19+12 > 23$ ✓

$A_\Delta = \sqrt{27(27-23)(27-19)(27-12)}$

$\boxed{A_\Delta = 113.842 \text{ units}^2}$

Ex#5 Find the area of $\triangle ABC$ given $a = 1.8\,cm$, $b = 5.1\,cm$, $C = 112°$.

$A_\Delta = \frac{ab\sin C}{2}$

$= \frac{1.8(5.1)\sin 112}{2}$

~~$A_\Delta = 8.512\,cm^2$~~

$\boxed{A_\Delta = 4.256\ cm^2}$