

12.1/12.2 Key Features of Quadratics/Parabolas and Word Problems

Standard Form: $y = ax^2 + bx + c$

Axis of symmetry: $x = -\frac{b}{2a}$	Vertex: $x = -\frac{b}{2a}$ (substitute x into $ax^2 + bx + c$ to get y -value)
x -intercepts: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	y -intercept: $(0, c)$

1. Write the following in standard form and identify the vertex.

a. $f(x) = (3x - 6)(x + 4)$
 $= 3x^2 + 12x - 6x - 24$

$f(x) = 3x^2 + 6x - 24$

$x = \frac{-6}{2(3)} = -1$

$y = 3(-1)^2 + 6(-1) - 24 = 3 - 6 - 24 = -27$

$V(-1, -27)$

b. $f(x) = 2(x - 6)(20 - 3x)$
 $= 2(20x - 3x^2 - 120 + 18x)$
 $= 2(-3x^2 + 38x - 120)$

$f(x) = -6x^2 + 76x - 240$

$x = \frac{-76}{-12} \approx 6.333$

$y = -6(6.333)^2 + 76(6.333) - 240 \approx 0.333$

$V(6.333, 0.333)$

2. Identify the x - and y -intercepts of the following:

a. $f(x) = x^2 + 11x + 30$

$x = \frac{-11 \pm \sqrt{11^2 - 4(1)(30)}}{2(1)} = \frac{-11 \pm \sqrt{121 - 120}}{2} = \frac{-11 \pm 1}{2}$

$\left\{ \begin{array}{l} \frac{-11-1}{2} = -6 \\ \frac{-11+1}{2} = -5 \end{array} \right.$

x -int: -6 and -5

y -int: $(0, 30)$

b. $f(x) = 4x^2 + 14x - 8$

$x = \frac{-14 \pm \sqrt{14^2 - 4(4)(-8)}}{2(4)} = \frac{-14 \pm \sqrt{324}}{8} = \frac{-14 \pm 18}{8}$

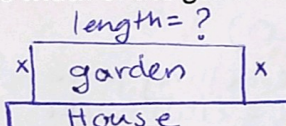
$\left\{ \begin{array}{l} \frac{-14+18}{8} \\ \frac{-14-18}{8} \end{array} \right.$

x -int: 0.5 and -4

y -int: $(0, -8)$

3. Mr. Picasso would like to create a small rectangular vegetable garden adjacent to his house. He has 24 ft of fencing to put around the three sides of his garden.

a. If the width of the garden is x feet, write an expression that represents the length of the garden.



$length = 24 - 2x$

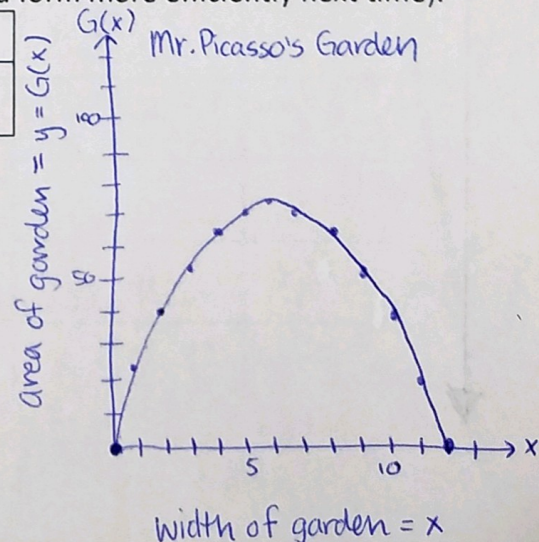
b. Write $G(x)$, the quadratic function in standard form representing the area of the garden.

Area of garden = $G(x) = length \cdot width = (24 - 2x)x$

$G(x) = -2x^2 + 24x$

c. Graph $G(x)$ using an x - y table (we'll learn to graph in standard form more efficiently next time).

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$G(x)$	0	22	40	54	64	70	72	70	64	54	40	22	0



d. What is the vertex? What do the coordinates of the vertex represent?

$$V(6, 72)$$

When the width of the garden is 6 ft, the area of the garden is 72 ft².

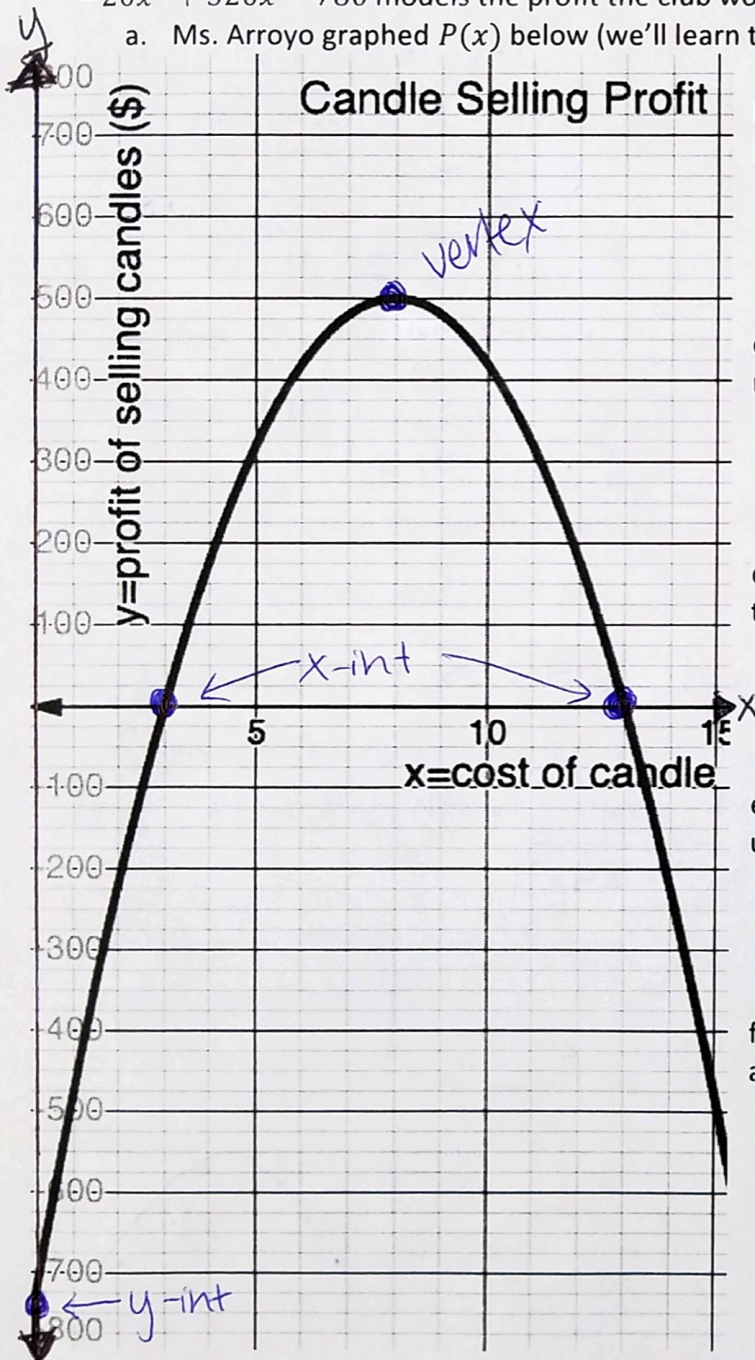
e. What are the dimensions of the garden that will have the maximum area? Why?

$$\text{max} = \text{vertex} = \text{width of } 6 \text{ ft} \Rightarrow \text{length} = 24 - 2(6) = 12 \text{ ft}$$

dimensions of max area are 6 ft x 12 ft

4. Mr. Picasso's art club students make and sell candles to raise money for art supplies. The function $P(x) = -20x^2 + 320x - 780$ models the profit the club would make by selling the candles for x dollars.

a. Ms. Arroyo graphed $P(x)$ below (we'll learn to graph in standard form next time).



b. What is the y-intercept and what does it represent in the situation?

$$y\text{-int: } (0, -780)$$

When the ~~cost~~^{selling price} of the candles is \$0, the club loses \$780.

c. What are the x-intercepts and what do they represent in the situation?

$$x\text{-int: } 3 \text{ and } 13$$

When the selling price of the candles is \$3 or \$13, the club makes \$0 (break even).

d. What is the vertex and what does it represent in this situation?

$$V(8, 500)$$

When the selling price of the candles is \$8, the club makes \$500 (max profit).

e. What are the units of the domain? What are the units of the range?

domain: selling price of candle in \$

range: profit of club in \$

f. What are a reasonable domain and range of $P(x)$, assuming the club does not want to lose money?

$$\text{domain: } [3, 13]$$

$$\text{range: } [0, 500]$$