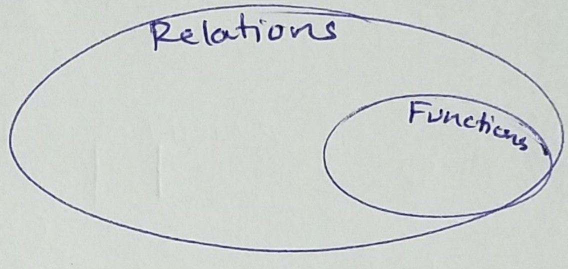
1.5 Inverses (& Parametric, but we won't do that)

Relation: Relates x & y-values, not always a function
(eg. $x^2+y^2=1$)
circle

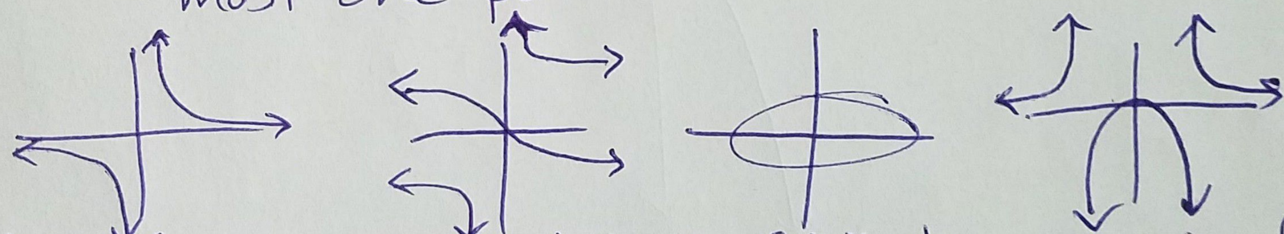
Function: One & only one y-val for every x-value.

☆ Inverse Relation: Switch the x & y-values.



Horizontal Line Test

The inverse of a relation is a function <u>iff</u> each horizontal line intersects the graph of the original relation in at most one point.



$f(x)$ is a function
inverse is a function
$f^{-1}(x)$ is a function

$f(x)$ is <u>not</u> a function
inverse <u>is</u> a function

$f(x)$ <u>is not</u> a function
inverse <u>is not</u> a function

$f(x)$ is a function
inverse <u>is not</u> a function

One-to-one

If the original function and the inverse function are <u>both</u> function (pass vert & horiz line tests) then the functions are one-to-one.

Inverse Function

$f^{-1}(x)$ "f inverse x"

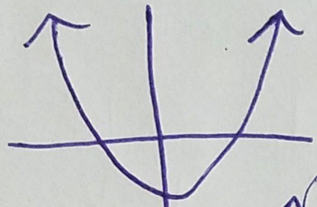
$f^{-1}(b)=a$ iff $f(a)=b$

Graphically

The graphs of f & f^{-1} are reflections over the line $y=x$. (Switch the x's & y's).

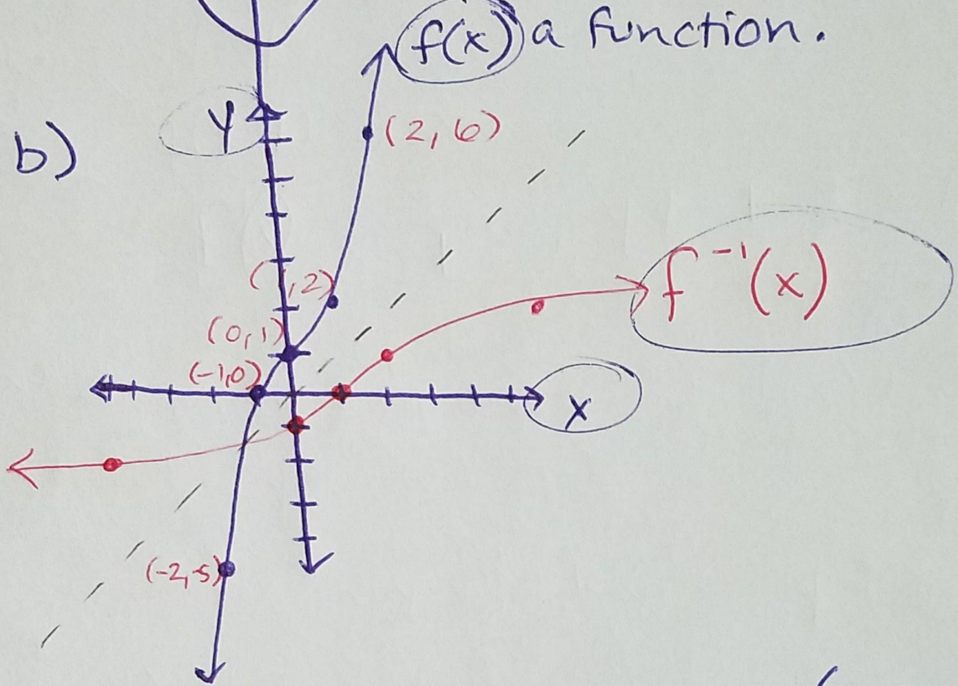
Ex #3 sketch the graph of the inverse function.

a)



Not one-to-one, therefore the the inverse is not a function.

b)



Proving Functions are Inverses (only way)

f & g are inverses iff

$f(g(x)) = x$ & $g(f(x)) = x$

Ex #4 Prove $f(x) = x^3+1$ & $g(x) = \sqrt[3]{x-1}$ are inverses.

$f(g(x)) = (\sqrt[3]{x-1})^3 + 1$

$= x - 1 + 1$

$f(g(x)) = x$ ✓

$g(f(x)) = \sqrt[3]{(x^3+1) - 1}$

$= \sqrt[3]{x^3}$

$g(f(x)) = x$ ✓

Since $f(g(x)) = x$ & $g(f(x)) = x$, then the functions f & g are inverses of eachother.

Ex # 1 Find the inverse of $f(x) = \frac{x}{x+1}$.

$$\lim_{x \to \infty} \frac{x}{x+1}$$

$$\lim_{x \to \infty} \frac{\frac{x}{x}}{\frac{x}{x} + \frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{1}{1 + \frac{1}{x}}$$

$$\frac{1}{1+0}$$

$$y \neq 1$$

$$y = \frac{x}{x+1} \qquad x \neq -1 \qquad y \neq 1$$

$$x = \frac{y}{y+1} \qquad y \neq -1 \qquad x \neq 1$$

$$x(y+1) = y$$

$$xy + x = y$$

$$xy - y = -x$$

$$y(x-1) = -x$$

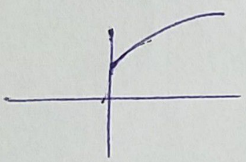
$$\boxed{\cancel{f(x)} = \frac{-x}{x-1}, \quad y \neq -1 \ \& \ x \neq 1}$$

$$f^{-1}(x)$$

Ex #2 Find the inverse of $f(x) = \sqrt{3x-2} + 1$.

$$3x - 2 \geq 0$$

$$x \geq \frac{2}{3} \qquad y \geq 1$$



$$y = \sqrt{3x-2} + 1$$

$$x = \sqrt{3y-2} + 1 \qquad y \geq \frac{2}{3} \qquad x \geq 1$$

$$x - 1 = \sqrt{3y-2}$$

$$(x-1)^2 = 3y - 2$$

$$(x-1)^2 + 2 = 3y$$

$$\frac{1}{3}(x-1)^2 + \frac{2}{3} = y$$

$$\boxed{f^{-1}(x) = \frac{1}{3}(x-1)^2 + \frac{2}{3}, \quad y \geq \frac{2}{3} \ \& \ x \geq 1}$$