24.2 (Part 2) Interest Formulas

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- r → RATE as a decimal
- t → TIME in years
- n → # of compounds per year
- P → initial value = PRINCIPAL
- A → total AMOUNT of $ in acct after t years

Compounding	Times per Year
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365
Annually	1

Ex #1 An account that pays 4% annual interest compounded quarterly has an initial investment of $3000. How much money will be in the account after three years?

$r = 0.04$
$n = 4$
$P = 3000$
$t = 3$
$A = ??$

$A = P(1 + \frac{r}{n})^{nt}$

$A = 3000\left(1 + \left(\frac{0.04}{4}\right)\right)^{4(3)}$

$A = 3380.475093959$

$3380.48

Ex #2 How long would it take an investment of $5000 to earn $1000 interest if it is invested in a savings account that pays 3.75% annual interest compounded monthly?

$t = ??$
$P = 5000$
$A = 6000$
$r = 0.0375$
$n = 12$

$A = P(1 + \frac{r}{n})^{nt}$

$\frac{6000}{5000} = \frac{5000}{5000}\left(1 + \frac{0.0375}{12}\right)^{12t}$

$1.2 = 1.003125^{12t}$

$\log_{1.003125} 1.2 = 12t$

$\frac{58.43401155}{12} = \frac{12t}{12}$

$t = 4.869500962$

4.870 years

Ex #3 A deposit of $1000 is made into a savings account that pays 4% annual interest compounded monthly.

a) How much money will be in the account after 6 years?

$P = 1000$
$r = .04$
$n = 12$
$t = 6$
$A = ??$

$A = P(1 + \frac{r}{n})^{nt} = 1000\left(1 + \frac{.04}{12}\right)^{12(6)} = 1270.741879079$

$1270.74

b) How long will it take for the $1000 to double?

$P = 1000$
$A = 2000$
$r = .04$
$n = 12$
$t = ??$

$\frac{2000}{1000} = \frac{1000}{1000}\left(1 + \frac{.04}{12}\right)^{12t}$

$2 = 1.003333333^{12t}$

$\log_{1.003333333} 2 = 12t$

$\frac{208.2905376}{12} = \frac{12t}{12}$

$t = 17.3575448$

$t =$ **17.358 years**

Ex #4 June invests $7500 at 12% interest for one year.

a) How much would she have if the interest is compounded yearly?

$P = 7500$
$r = .12$
$t = 1$
$n = 1$

$A = 7500\left(1 + \frac{.12}{1}\right)^{1(1)}$

$A = 8400$

$8400

b) How much would she have if the interest is compounded daily?

$P = 7500$
$r = .12$
$t = 1$
$n = 365$

$A = 7500\left(1 + \frac{.12}{365}\right)^{365(1)}$

$A = 8456.059617288$

$8456.06

Continually Compounded Interest Formula

$A = Pe^{rt}$ → r: RATE as a decimal; t: TIME in years; e: button on calculator ≈ 2.7; P: PRINCIPAL; A: AMOUNT/total

Ex #5 An account that pays 4% annual interest compounded continuously has an initial investment of $3000. How much money will be in the account after three years?

$r = .04$, $P = 3000$, $t = 3$, $A = ??$

$A = Pe^{rt}$

$A = 3000e^{.04(3)}$

$A = 3382.4905547381$

$3382.49

Ex #6 How long would it take an investment of $5000 to earn $1000 interest if it is invested in a savings account that pays 3.75% annual interest compounded continuously?

$t = ??$, $P = 5000$, $A = 6000$, $r = .0375$

$A = Pe^{rt}$

$\frac{6000}{5000} = \frac{5000e^{.0375t}}{5000}$

$1.2 = e^{.0375t}$

$\ln 1.2 = .0375t$

$.0375t = 0.182321556$

$t =$ **4.862 years**

Ex #7 If $4000 is invested at 7% interest per year compounded continuously, how long will it take to double the original investment?

$8000 = 4000e^{.07t}$

$2 = e^{.07t}$

$\ln 2 = .07t$

$t = \frac{\ln 2}{.07}$

9.902 years

Ex #8 A deposit of $4000 is made into a savings account that pays 2.48% annual interest compounded quarterly

a) How much money will be in the account after three years?

$A = 4000(1 + \frac{.0248}{4})^{4(3)}$

$A = 4307.9608435887$

$4307.96

b) How long will it take for the account to earn $500 interest?

$4500 = 4000(1 + \frac{.0248}{4})^{4t}$

$1.125 = 1.0062^{4t}$

$4t = \log_{1.0062} 1.125 \rightarrow t = \frac{\log_{1.0062} 1.125}{4}$

4.764 years

c) How much more money will be in the account after three years if the interest is compounded continuously?

$A = 4000e^{.0248(3)}$

$A = 4308.9564574978$

$4308.95