Conner

6.1/6.2

Integration by Parts

$\int e^x dx = e^x + C$:)

$\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$:)

$\int x e^{6x} dx$:(Int. by parts.

Where formula comes from:

$(fg)' = f'g + fg'$

$\int (fg)' dx = \int f'g\,dx + \int fg'\,dx$

$fg = \int f'g\,dx + \int fg'\,dx$

$\int fg'\,dx = fg - \int f'g\,dx$

Let $f(x) = u \quad f'(x)dx = du$

$g(x) = v \quad g'(x)dx = dv$

$\boxed{\int u\,dv = uv - \int v\,du}$ "ultraviolet super voodoo"

Ex #1 $\int x e^{6x} dx = \frac{x}{6} e^{6x} - \frac{1}{6}\int e^{6x} dx$ $\qquad \bullet\ u = x \qquad v = \frac{1}{6} e^{6x}$

$= \boxed{\frac{x}{6} e^{6x} - \frac{1}{36} e^{6x} + C}$ $\qquad du = dx \qquad \bullet\ dv = e^{6x} dx$

Ex #2 $\int (3x+5)\cos\left(\frac{x}{4}\right) dx$ $\qquad \bullet\ u = 3x+5 \qquad v = 4\sin\left(\frac{x}{4}\right)$

$= 4(3x+5)\sin\left(\frac{x}{4}\right) - 12\int \sin\left(\frac{x}{4}\right) dx$ $\qquad du = 3dx \qquad \bullet\ dv = \cos\left(\frac{x}{4}\right) dx$

$= 4(3x+5)\sin\left(\frac{x}{4}\right) + 12\left(4\cos\left(\frac{x}{4}\right)\right) + C$

$= \boxed{4(3x+5)\sin\left(\frac{x}{4}\right) + 48\cos\left(\frac{x}{4}\right) + C}$

Ex #3 $\int x^2 \sin(10x) dx = \frac{-x^2}{10}\cos(10x) + \frac{1}{5}\int x\cos(10x) dx$ $\qquad \bullet\ u = x^2 \qquad v = \frac{-1}{10}\cos(10x)$

$du = 2x\,dx \qquad \bullet\ dv = \sin(10x) dx$

$\bullet\ u = x \qquad v = \frac{1}{10}\sin(10x)$

$du = dx \qquad \bullet\ dv = \cos(10x) dx$

$\int x^2 \sin(10x) dx = -\frac{x^2}{10}\cos(10x) + \frac{1}{5}\left[\frac{x}{10}\sin(10x) - \frac{1}{10}\int \sin(10x) dx\right]$

$\int x^2 \sin(10x) dx = \boxed{-\frac{x^2}{10}\cos(10x) + \frac{x}{50}\sin(10x) + \frac{1}{500}\cos(10x) + C}$

Ex #4 $\int e^x \cos x\,dx = e^x\cos x + \int e^x \sin x\,dx$ $\qquad u = \cos x \qquad v = e^x$

$\int e^x \cos x\,dx = e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$ $\qquad du = -\sin x\,dx \qquad dv = e^x dx$

$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x + C$ $\qquad u = \sin x \qquad v = e^x$

$\boxed{\int e^x \cos x\,dx = \frac{e^x}{2}(\cos x + \sin x) + C}$ $\qquad du = \cos x\,dx \qquad dv = e^x dx$

Definite Integrals

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Partial Fractions

Degree of numerator <u>must</u> be less than denominator's.
Otherwise use polynomial long division!

factor in denominator	term in partial fraction decomposition
$(ax+b)^k$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \cdots + \frac{Z}{(ax+b)^k}$
$(ax^2+bx+c)^k$	$\frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2} + \cdots + \frac{Yx+Z}{(ax^2+bx+c)^k}$

Ex #5 $\int \frac{3x+11}{x^2-x-6}\,dx$

$$\left[\frac{3x+11}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}\right](x-3)(x+2)$$

$$3x+11 = A(x+2) + B(x-3)$$

$$3x+11 = Ax+2A+Bx-3B$$

$$3x+11 = (A+B)x + 2A-3B$$

$3 = A+B$ $\qquad$ $11 = 2A-3B$

$A=4$ $\qquad$ $B=-1$

$$\int \frac{3x+11}{x^2-x-6}\,dx = 4\int \frac{1}{x-3}\,dx - 1\int \frac{1}{x+2}\,dx$$

$$= \boxed{4\ln|x-3| - \ln|x+2| + C}$$

Ex #6 $\int \frac{x^2+4}{3x^3+4x^2-4x}\,dx$

$$\frac{x^2+4}{x(x+2)(3x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{3x-2}$$

$$x^2+4 = A(x+2)(3x-2) + Bx(3x-2) + Cx(x+2)$$

$$x^2+4 = 3Ax^2+4Ax-4A+3Bx^2-2Bx+Cx^2+2Cx$$

$1 = 3A+3B+C$ $\qquad$ $0 = 4A-2B+2C$ $\qquad$ $4=-4A$

$A=-1$ $\qquad$ $B=\frac{1}{2}$ $\qquad$ $C=\frac{5}{2}$

$$\int \frac{x^2+4}{3x^3+4x^2-4x}\,dx = -1\int \frac{1}{x}\,dx + \frac{1}{2}\int \frac{1}{x+2}\,dx + \frac{5}{2}\int \frac{1}{3x-2}\,dx$$

$$= -\ln|x| + \frac{1}{2}\ln|x+2| + \frac{5}{2}\left(\frac{1}{3}\right)\ln|3x-2| + C$$

$$= \boxed{-\ln|x| + \frac{1}{2}\ln|x+2| + \frac{5}{6}\ln|3x-2| + C}$$

Ex#7 $\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx$

$$\frac{x^2-29x+5}{(x-4)^2(x^2+3)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$$

$$x^2-29x+5 = A(x-4)(x^2+3) + B(x^2+3) + (Cx+D)(x-4)^2$$

$$x^2-29x+5 = Ax^3-4Ax^2+3Ax-12A+Bx^2+3B+Cx^3+Dx^2-8Cx^2-8Dx+16Cx+16D$$

$0=A+C \qquad 1=-4A+B-8C+D \qquad -29=3A+16C-8D \qquad 5=-12A+3B+16D$

$$[A]=\begin{bmatrix} 1 & 0 & 1 & 0 \\ -4 & 1 & -8 & 1 \\ 3 & 0 & 16 & -8 \\ -12 & 3 & 0 & 16 \end{bmatrix} \qquad [B]=\begin{bmatrix} 0 \\ 1 \\ -29 \\ 5 \end{bmatrix} \qquad A^{-1}B=\begin{bmatrix} 1 \\ -5 \\ -1 \\ 2 \end{bmatrix}$$

$A=1 \qquad B=-5 \qquad C=-1 \qquad D=2$

$$\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}\,dx = 1\int\frac{1}{x-4}\,dx - 5\int\frac{1}{(x-4)^2}\,dx + \int\frac{-x+2}{x^2+3}\,dx$$

$$= \ln|x-4| + \frac{5}{x-4} - \int\frac{x}{x^2+3}\,dx + 2\int\frac{1}{x^2+3}\,dx$$

$$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + 2\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

$$= \boxed{\ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$

Ex#8 $\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx$

$$\begin{array}{r} x-2-\dfrac{18}{x^3-3x^2} \\ x^3-3x^2 \overline{\big)\, x^4-5x^3+6x^2+0x-18} \\ -(x^4-3x^3+0x^2+0x) \\ \hline -2x^3+6x^2+0x-18 \\ -(-2x^3+6x^2+0x+0) \\ \hline -18 \end{array}$$

⋮

$$\int\frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx = \frac{1}{2}x^2-2x+2\ln|x|-\frac{6}{x}-2\ln|x-3|+C$$