2.6 Graphs of Rational Functions

Rational Function

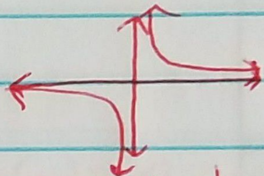
Given the polynomial functions f & g w/ $g(x) \neq 0$ then

$$r(x) = \frac{f(x)}{g(x)}$$

is a rational function.

Domain → determine what will make the denominator undefined & all else is in domain.

Transformations



$$f(x) = \frac{a}{x-h} + k$$

a → vertical stretch $a > 1$ or $a < -1$

shrink/compression $-1 < a < 1$

reflection over x-axis.

h → horizontal translation (do opposite)

k → vertical translation

Ex #1 $f(x) = \frac{3x-7}{x-2}$ → Describe the transformations.

$$\begin{array}{r} 3 \\ x-2 \overline{)\,3x-7} \\ -(\cancel{3x-6}) \\ \hline -1 \end{array}$$

$$\frac{3x-7}{x-2} = 3 - \frac{1}{x-2}$$

$$f(x) = \frac{-1}{x-2} + 3$$

- reflection over the x-axis
- horizontal translation of 2 units right
- vertical translation of 3 units up.

$f(x) = \frac{3}{x+2}$ · vertical stretch by a factor of 3

Asymptotes

VA: Determine when (for what values) the denom $= 0$ $\quad x = \#,\ x = \#$

HA: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x) = y \leftarrow \underline{HA}$ $\quad y = \#$

SA: undefined limit $\rightarrow$ long division to det. Slant asymp.

Ex #2 Determine the asymptotes of the following & use limit notation when appropriate.

a) $f(x) = \frac{3x-7}{x-2}$

b) $f(x) = \frac{x-1}{x^2-x-6}$

c) $f(x) = \frac{x^3}{x^2-9}$

a) VA: $x - 2 = 0$ $\quad \boxed{x = 2}$ $\quad \boxed{\lim_{x\to 2^-} f(x) = \infty \quad \lim_{x\to 2^+} f(x) = -\infty}$

HA: $\lim_{x\to\infty} \frac{3x-7}{x-2} = \lim_{x\to\infty} \frac{3 - 7/x}{1 - 2/x} = 3/1 = 3$ $\quad \boxed{y = 3}$

$\lim_{x\to-\infty} \frac{3 - \cancel{7/x}}{1 - \cancel{2/x}} = 3$ $\quad \boxed{\lim_{x\to\pm\infty} f(x) = 3}$

b) VA: $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3,\ x = -2}$

$\boxed{\lim_{x\to 3^-} f(x) = -\infty \quad \lim_{x\to 3^+} f(x) = \infty}$ $\quad \lim_{x\to -2^-} f(x) = -\infty \quad \lim_{x\to 2^+} f(x) = \infty$

HA: $\lim_{x\to\infty} \frac{x-1}{x^2-x-6} = \lim_{x\to\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{6}{x^2}} = \frac{0}{1} = 0$ $\quad \boxed{y = 0}$

$\boxed{\lim_{x\to\pm\infty} f(x) = 0}$

c) VA: $x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$\boxed{x = 3,\ x = -3}$

$\boxed{\lim_{x\to 3^-} f(x) = -\infty \quad \lim_{x\to -3^-} f(x) = -\infty \quad \lim_{x\to 3^+} f(x) = \infty \quad \lim_{x\to -3^+} f(x) = \infty}$

HA: $\lim_{x\to\infty} \frac{x^3}{x^2-9} = \lim_{x\to\infty} \frac{1}{\frac{1}{x} - \frac{9}{x^3}} = \frac{1}{0} \rightarrow \underline{\text{undef}}$ $\quad$ NO HA

X

SA:

~~$x^3 \overline{)x^2 + 0x - 9}$~~

$x^2 + 0x - 9 \overline{)x^3 + 0x^2 + 0x + 0}$

$-(x^3 + 0x^2 - 9x)$

$9x \leftarrow R$ does not matter!

SA $\rightarrow \boxed{y = x}$

Ex #3 Graph $f(x) = \frac{x^2-x-2}{x+2}$ $\cancel{(x+2)}\,0 = \frac{x^2-x-2}{\cancel{x+2}}\,\cancel{(x+2)}$

$0 = x^2-x-2$

① ID asymptotes

② ID intercepts (x & y)

③ make a table?

VA: $\boxed{x=-2}$ $0 = x+2$

HA/SA: NO HA! $\boxed{y = x-3}$

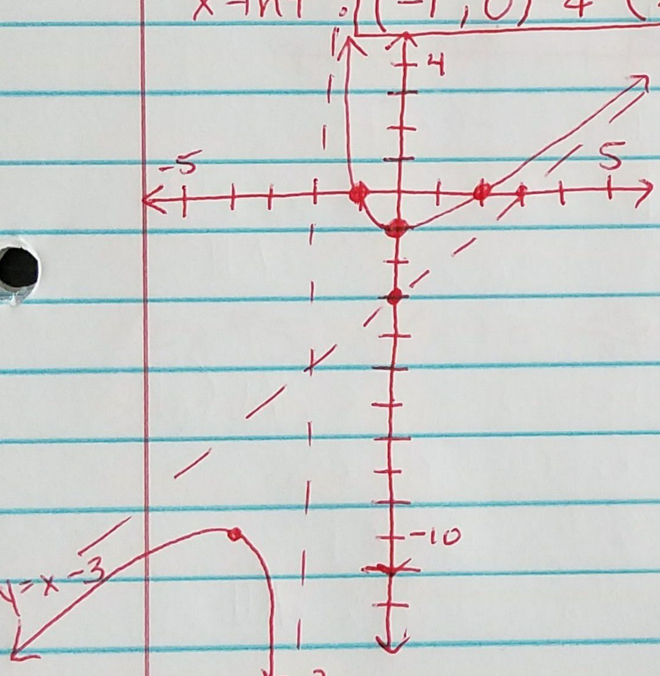
$$\begin{array}{r} x-3 \\ x+2\,\overline{)\,x^2-x-2} \\ -(x^2+2x) \\ \hline -3x-2 \end{array}$$

y-int: $\boxed{(0,-1)}$ $f(0) = -1$

x-int: $\boxed{(-1,0) \ \& \ (2,0)}$ ⟵ $0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x=2,\ x=1$



x	y
-3	-10

$f(-3) = -10$

Holes in the Graph

Has a removable discontinuity.

This happens when factors cancel out.

Ex #4 Graph $f(x) = \frac{4x^2+x}{2x^2+x}$

$= \frac{\cancel{x}(4x+1)}{\cancel{x}(2x+1)}$ hole @ x=0

$f(x) = \frac{4x+1}{2x+1}$

VA: $2x+1=0$ $\boxed{x=-\frac{1}{2}}$

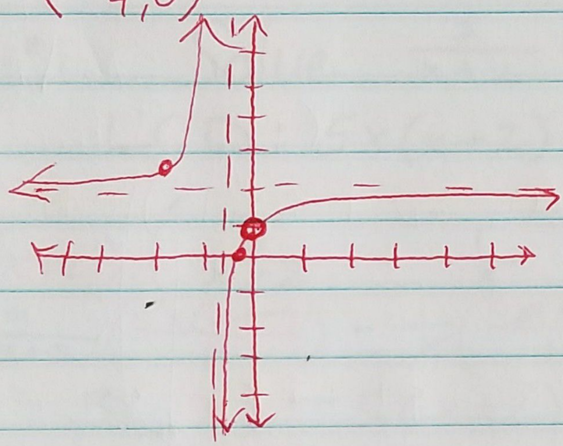
HA: $\lim_{x\to\infty} \frac{4x+1}{2x+1} = \lim_{x\to\infty} \frac{4+1/x}{2+1/x} = 2$ $\boxed{y=2}$

y-int: $(0, 1)$ ← since x=0 here it is a hole

x-int: $0=4x+1$ $x=-\frac{1}{4}$

$(-\frac{1}{4}, 0)$

$f(-2) = \frac{-8+1}{-4+1} = \frac{-7}{-3} \approx 2.3\overline{3}$



Ex #5 $f(x) = \frac{2x^2+5}{x^2-25}$ Graph