20.2/20.3 Geometric Series

Finite Sum of a Geometric

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Ex #1 Find S_8 for the geometric series with $a_1=5$ and $r=2$.

you must know how to put into calculator

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $\quad 5+10+20+40+80+160+320+640$

$S_8 = 5\left(\frac{1-2^8}{1-2}\right)$ $\quad \boxed{S_8 = 1275}$

Ex #2 $\quad 256+64+16+4+\cdots$ $\quad$ Find S_6.

$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $\quad 256+64+16+4+1+\frac{1}{4}$

$S_6 = 256\left(\frac{1-(\frac{1}{4})^6}{1-(\frac{1}{4})}\right)$ $\quad \boxed{S_6 = 341.25}$

calculator ⇒ 256((1-(1/4)^6)/(1-(1/4)))

Ex #3 $\quad \sum_{n=1}^{10} \underbrace{2(3)^{n-1}}_{a_1(r)^{n-1}}$ ~~this is geometric~~

$a_1 = 2 \quad r = 3 \quad n = 10$

$S_{10} = 2\left(\frac{1-3^{10}}{1-3}\right)$ $\quad \boxed{S_{10} = 59{,}048}$

Infinite Sum of a Geometric

$$S = \frac{a_1}{1-r}$$

☆ only when $|r| < 1$ ☆

Ex #4 $\quad 64+16+4+1+\cdots$

$a_1 = 64 \quad r = \frac{1}{4}$

$S = \frac{64}{1-\frac{1}{4}}$ = 64/((1-(1/4))

$\boxed{S = 85.33333333 = 85.\overline{3} = 85\tfrac{1}{3}}$

Ex #5 $\quad \frac{1}{3}+\frac{5}{12}+\frac{25}{48}+\frac{125}{192}+\cdots$

$a_1 = \frac{1}{3}$ $\quad r = \frac{5}{4}$ $\quad$ $\boxed{\text{cannot calculate sum}}$

Ex #6 $\quad \sum_{n=1}^{\infty} \underbrace{3\left(\frac{2}{5}\right)^{n-1}}_{a_1(r)^{n-1}}$ $\quad a_1 = 3 \quad r = \frac{2}{5}$

$S = \frac{3}{1-\frac{2}{5}} = 5$ $\quad \boxed{S = 5}$