5.1 Fundamental Identities

Basic Trig Identities

Reciprocal Identities

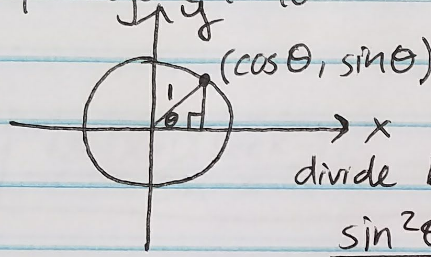
$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities



$\boxed{\sin^2\theta + \cos^2\theta = 1}$

or $(\sin\theta)^2 + (\cos\theta)^2 = 1$

divide by $\sin^2\theta$ to both sides

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

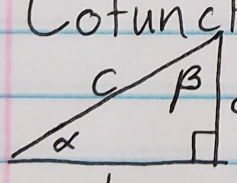
$\boxed{1 + \cot^2\theta = \csc^2\theta}$

divide by $\cos^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$

Cofunction Identities



α & β are complementary $(\alpha+\beta=90°)$

$\sin\alpha = \frac{a}{c}$ $\quad \cos\alpha = \frac{b}{c}$

$\sin\beta = \frac{b}{c}$ $\quad \cos\beta = \frac{a}{c}$

The sine of any angle is the same value as the cofunction of the complementary angle.

$\sin(\frac{\pi}{2}-\theta) = \cos\theta$ $\quad \tan(\frac{\pi}{2}-\theta) = \cot\theta$ $\quad \sec(\frac{\pi}{2}-\theta) = \csc\theta$

$\cos(\frac{\pi}{2}-\theta) = \sin\theta$ $\quad \cot(\frac{\pi}{2}-\theta) = \tan\theta$ $\quad \csc(\frac{\pi}{2}-\theta) = \sec\theta$

Ex #4 Simplify $\frac{(\sec x+1)(\sec x-1)}{\sin^2 x}$.

[Hint: If you're stuck try to write it in terms of sine & cosine]

$$\frac{(\sec x+1)(\sec x-1)}{\sin^2 x} = \frac{\sec^2 x - 1}{\sin^2 x}$$

$1+\tan^2 x = \sec^2 x$
$-1 \qquad\qquad -1$
$\tan^2 x = \sec^2 x - 1$

$$= \frac{\tan^2 x}{\sin^2 x}$$

$$= \frac{\cancel{\sin^2 x}}{\cos^2 x}\left(\frac{1}{\cancel{\sin^2 x}}\right)$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{\frac{(\sec x+1)(\sec x-1)}{\sin^2 x} = \sec^2 x}$$

Ex #5 Simplify $\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x}$

[Hint: One fraction is better than 2.]

$$\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x}{\cos x(1-\sin x)} - \frac{\sin x(1-\sin x)}{\cos x(1-\sin x)}$$

$$= \frac{\cos^2 x - \sin x + \sin^2 x}{\cos x(1-\sin x)}$$

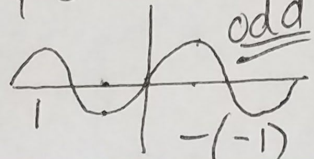
$$= \frac{(\sin^2 x + \cos^2 x) - \sin x}{\cos x(1-\sin x)}$$

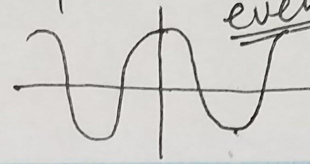
$$= \frac{\cancel{1-\sin x}}{\cos x\cancel{(1-\sin x)}}$$

$$= \frac{1}{\cos x}$$

$$\boxed{\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \sec x}$$

$y = \sin x$ is odd $\qquad y = \cos x$ is even





Odd-Even Identities

$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$

$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x$

$\cot(-x) = -\cot x$

$\tan(-x) = -\tan x$

Ex #1 Find $\sin\theta$ & $\cos\theta$ if $\tan\theta = 5$ & $\cos\theta > 0$ using only identities. [Hint: always start w/ Pyth. Id]

$1 + \tan^2\theta = \sec^2\theta$

$1 + 5^2 = \sec^2\theta$

$26 = \sec^2\theta$

$\pm\sqrt{26} = \sec\theta$

$\pm\sqrt{26} = \frac{1}{\cos\theta}$

$\pm\frac{1}{\sqrt{26}} = \cos\theta$

$\left(\frac{\sqrt{26}}{\sqrt{26}}\right)\frac{1}{\sqrt{26}} = \cos\theta$

$\frac{\sqrt{26}}{26} = \cos\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$5 = (\sin\theta)\left(\frac{26}{\sqrt{26}}\right)$

$\frac{5\sqrt{26}}{26} = \sin\theta$

$$\boxed{\begin{aligned} \sin\theta &= \frac{5\sqrt{26}}{26} \\ \cos\theta &= \frac{\sqrt{26}}{26} \end{aligned}}$$

Ex #2 Find $\sin(\theta - \frac{\pi}{2})$ if $\cos\theta = 0.340$. [Hint: This screams cofunctions!]

$\sin(\theta - \frac{\pi}{2}) = -\sin(\frac{\pi}{2} - \theta)$

$= -\cos\theta$

$$\boxed{\sin(\theta - \tfrac{\pi}{2}) = -0.340}$$

Ex #3 Simplify $\sin^3 x + \sin x\cos^2 x$. [Hint: powers greater than 2 usually mean you need to factor.]

$\sin^3 x + \sin x\cos^2 x = \sin x(\sin^2 x + \cos^2 x)$

$= \sin x(1)$

$$\boxed{\sin^3 x + \sin x\cos^2 x = \sin x}$$