5.1 (con't)   Solving Trig Equations

Things to keep in mind
- trig identities   • reference triangles/angles
- algebraic solving methods (factoring, quadratic formula, substitution, etc.)
- unit circle   • parent graphs
- ~~inv~~ inverse trig functions • domain restrictions

Ex #6   Solve $\sin x + 2 = 3$ over the interval $0 \le x \le 2\pi$.

$$\sin x + 2 = 3$$
$$\sin x = 1$$
$$x = \sin^{-1} 1$$
$$\boxed{x = \frac{\pi}{2}}$$

Ex #7   Solve $\tan^2 x + 3 = 0$ for $x \in [0, 2\pi]$.

$$\tan^2 x + 3 = 0$$
$$\tan^2 x = -3$$
$$\tan x = \pm\sqrt{-3}$$
$$\boxed{\text{No solution}}$$

Ex #8 Solve $2\cos^2 x - \sqrt{3}\cos x = 0$
for $x \in [0°, 360°]$.

$$2\cos^2 x - \sqrt{3}\cos x = 0$$
$$\cos x(2\cos x - \sqrt{3}) = 0$$

$\cos x = 0$ or $2\cos x - \sqrt{3} = 0$

$x = \cos^{-1} 0$ $2\cos x = \sqrt{3}$

$\rightarrow x = 90°$ $\cos x = \frac{\sqrt{3}}{2}$

$\rightarrow x = 270°$ $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = 30°, 330°$

$$\boxed{x = 90°, 270°, 30°, 330°}$$
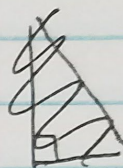
Ex #9 Solve $\sin^2 x - \sin x = 2$.

$$\sin^2 x - \sin x - 2 = 0$$
Let $y = \sin x$.
$$y^2 - y - 2 = 0$$
$$(y - 2)(y + 1) = 0$$

$y - 2 = 0$ or $y + 1 = 0$

$y = 2$ $y = -1$

$\sin x = 2$ $\sin x = -1$

$x = \sin^{-1} 2$ $x = \sin^{-1}(-1)$

No solution $\rightarrow x = -\frac{\pi}{2} + 2\pi n$

also correct $\boxed{x = \frac{3\pi}{2} + 2\pi n}$
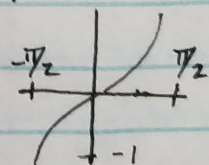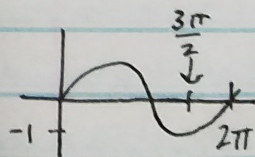
EX #10 Solve $\cos^2 x + \cos x = \sin^2 x$. Give answer in radians.

$$\cos^2 x + \cos x = \sin^2 x$$
$$\cos^2 x + \cos x = 1 - \cos^2 x$$
$$2\cos^2 x + \cos x - 1 = 0$$
Let $y = \cos x$.
$$2y^2 + y - 1 = 0$$
$$(2y - 1)(y + 1) = 0$$

$2y - 1 = 0$ $\qquad\qquad$ $y + 1 = 0$
$2y = 1$ $\qquad\qquad\qquad$ $y = -1$
$y = \frac{1}{2}$ $\qquad\qquad\qquad$ $\cos x = -1$
$\cos x = \frac{1}{2}$ $\qquad\qquad\quad$ $x = \cos^{-1}(-1)$
$x = \cos^{-1}\left(\frac{1}{2}\right)$ $\qquad\qquad$ $x = \pi$
$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$$\boxed{\begin{array}{l} x = \frac{\pi}{3} + 2\pi n \\ x = \frac{5\pi}{3} + 2\pi n \\ x = \pi + 2\pi n \end{array}}$$