A30: pg. 275 # 8, 10-13

17.2 Fundamental Theorem of Algebra

Fundamental Theorem of Algebra (FToA)

A polynomial of degree n has exactly n linear factors and n zeros (roots/x-int/solutions), including those that show up multiple times.

Ex #1 Factor, then use the Zero Product Property, and show that the FTOA is true by counting the # of factors & zeros.

a) $f(x) = x^3 + 9x$

$= x(x^2 + 9)$

$3i \quad 9 \quad -3i \quad 0$

$0 = x(x + 3i)(x - 3i)$

$\boxed{x = 0 \quad x = -3i \quad x = 3i}$

$\boxed{\text{degree} = 3,\ 3 \text{ factors},\ 3 \text{ zeros}}$

b) $g(x) = x^4 - 16$

$4 \quad -16 \quad -4 \quad 0$

$= (x^2 + 4)(x^2 - 4)$

$2i \quad 4 \quad -2i \quad 0 \qquad 2 \quad -4 \quad -2 \quad 0$

$0 = (x + 2i)(x - 2i)(x + 2)(x - 2)$

$\boxed{x = -2i \quad x = 2i \quad x = -2 \quad x = 2}$

$\boxed{\text{degree is } 4,\ 4 \text{ factors, } \& \ 4 \text{ zeros}}$

c) $f(x) = x^3 - 64$

$= x^3 - 4^3$

$0 = (x - 4)(x^2 + 4x + 16)$

$16 \quad '' \quad '' \quad 4$

$\boxed{x = 4}$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(16)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16 - 64}}{2}$

$x = \dfrac{-4 \pm \sqrt{-48}}{2}$

$x = \dfrac{-4 \pm i\sqrt{48}}{2}$

$\boxed{x = \dfrac{-4 + i\sqrt{48}}{2} \quad x = \dfrac{-4 - i\sqrt{48}}{2}}$

$\left(x - \dfrac{-4 + i\sqrt{48}}{2}\right) \ \& \ \left(x - \dfrac{-4 - i\sqrt{48}}{2}\right)$

$\boxed{\begin{array}{l}\text{degree} = 3 \\ 3 \text{ factors} \\ 3 \text{ zeros } \checkmark\end{array}}$

Complex Conjugate Root Theorem

If $a+bi$ (or $a+b\sqrt{c}$) is a zero, then $a-bi$ (or $a-b\sqrt{c}$) is a zero, too!

Ex #2 Write a polynomial of n^{th} degree that has the given zeros in standard form.

a) $n=3$; $x=0$, $x=5$, $x=-7$

$$f(x)=(x-0)(x-5)(x-(-7))$$
$$=x(x-5)(x+7)$$
$$=x(x^2+2x-35)$$
$$\boxed{f(x)=x^3+2x^2-35x}$$

	x	-5
x	x^2	$-5x$
$+7$	$7x$	-35

b) $n=4$; $x=3$, $x=-3$, $x=1+2i$, $x=1-2i$

$$f(x)=(x-3)(x+3)(x-(1+2i))(x-(1-2i))$$

	x	-3
x	x^2	$\cancel{-3x}$
$+3$	$\cancel{3x}$	-9

	x	-1	$-2i$
x	x^2	$-x$	$\cancel{-2xi}$
-1	$-x$	1	$\cancel{2i}$
$+2i$	$\cancel{2xi}$	$\cancel{-2i}$	$\cancel{-4i^2}$ 4

$$f(x)=(x^2-9)(x^2-2x+5)$$

	x^2	-9
x^2	x^4	$-9x^2$
$-2x$	$-2x^3$	$18x$
$+5$	$5x^2$	-45

$$\boxed{f(x)=x^4-2x^3-4x^2+18x-45}$$

c) $n=4$; $x=2$, $x=-5$, $x=-4$ (double root)

$$f(x)=(x-2)(x+5)(x+4)(x+4)$$
$$\boxed{f(x)=x^4+11x^3+30x^2-32x-160}$$