Name: Arroyo Date: 8/20 Period: 3

Notes 1.2: Functions and their Properties

Vocabulary

Function: A relation where every x-value is paired with a unique (one & only one) y-value.

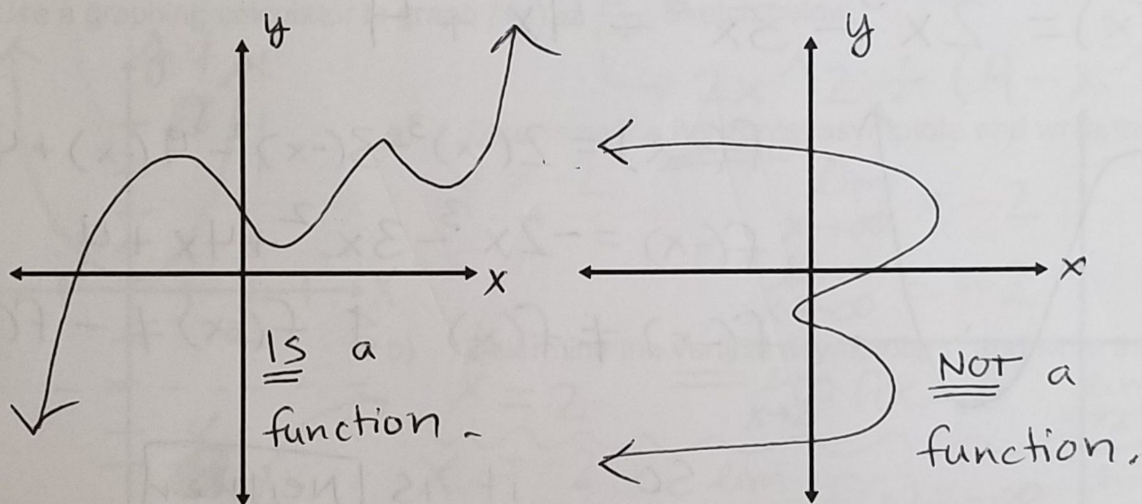
Domain: The set of all possible inputs; x-values.

Range: The set of all possible outputs; y-values.

Vertical Line Test

A graph on the xy-plane defines $f(x) = y$ as a function iff (if and only if) ... no vertical line intersects the graph in more than one point.

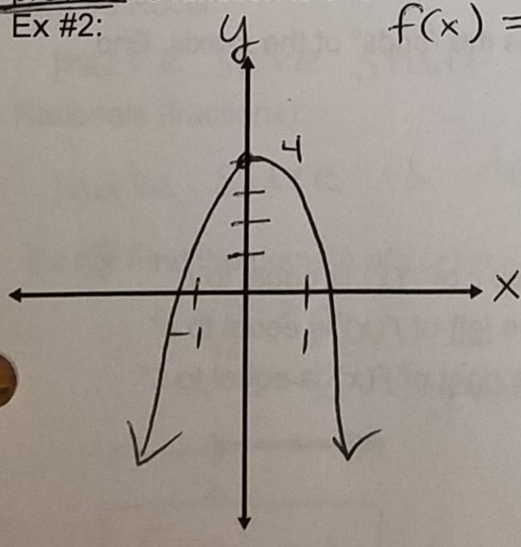
Ex #1:



Even and Odd Functions

Graphically, even functions are symmetric about the y-axis. Algebraically, showing $f(-x) = f(x)$ proves the function is even.

Ex #2:



$f(x) = -3x^2+4$

$f(-x) = -3(-x)^2+4$

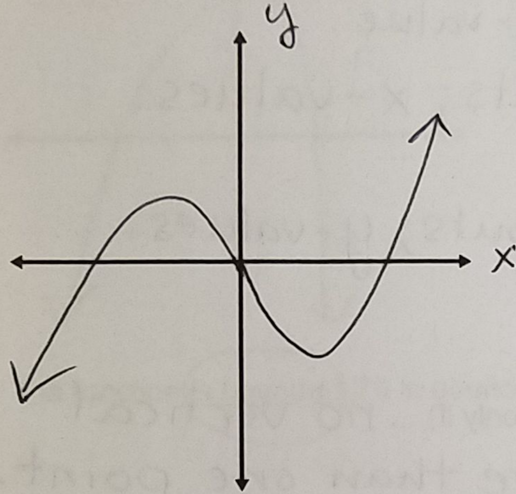
$f(-x) = -3x^2+4$

$f(-x) = f(x)$

therefore, the function is $\boxed{\text{Even!}}$

Graphically, odd functions are symmetric about the origin. Algebraically, showing $f(-x) = -f(x)$ proves the function is odd.

Ex #3: $f(x) = 2x^3 - 4x$



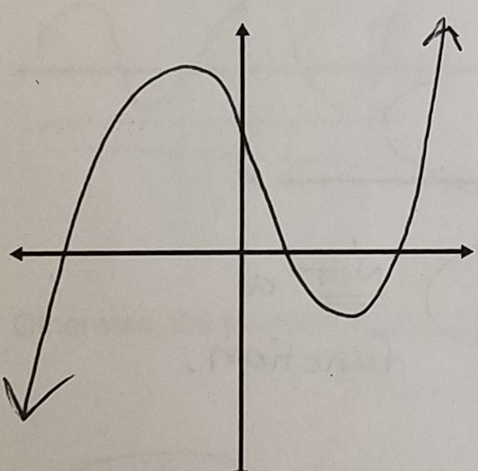
$f(-x) = 2(-x)^3 - 4(-x)$

$f(-x) = -2x^3 + 4x$

$f(-x) = -f(x)$

$\boxed{\text{odd}}$

Ex #4: $f(x) = 2x^3 - 3x^2 - 4x + 4$



$f(-x) = 2(-x)^3 - 3(-x)^2 - 4(-x) + 4$

$f(-x) = -2x^3 - 3x^2 + 4x + 4$

$f(-x) \neq f(x)$ & $f(-x) \neq -f(x)$

so it is $\boxed{\text{Neither}}$

End Behavior

Tells us how the function (y-values) behaves as it goes off towards the "ends" of the x-axis. End behavior asks us to determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

Limit Notation

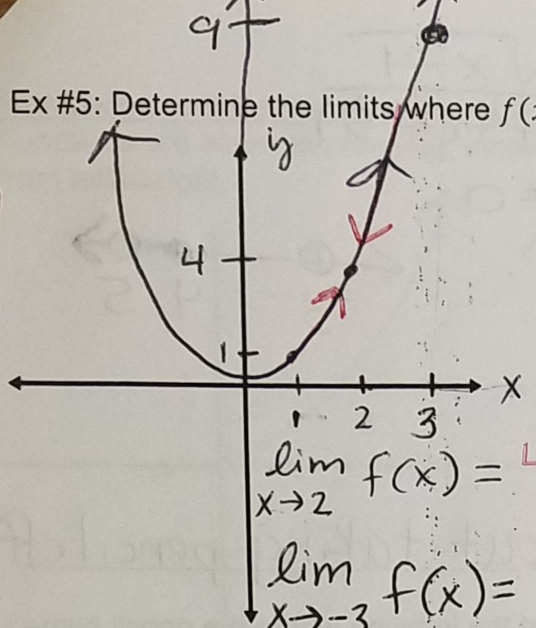
(AKA: as $x \to c, f(x) \to L$)

$\lim_{x\to c} f(x) = L$ "the limit as x approaches c of $f(x)$ is equal to L"

$\lim_{x\to c^-} f(x) = L$ "the limit as x approaches c from the <u>left</u> of $f(x)$ is equal to L"

$\lim_{x\to c^+} f(x) = L$ "the limit as x approaches c from the <u>right</u> of $f(x)$ is equal to L"

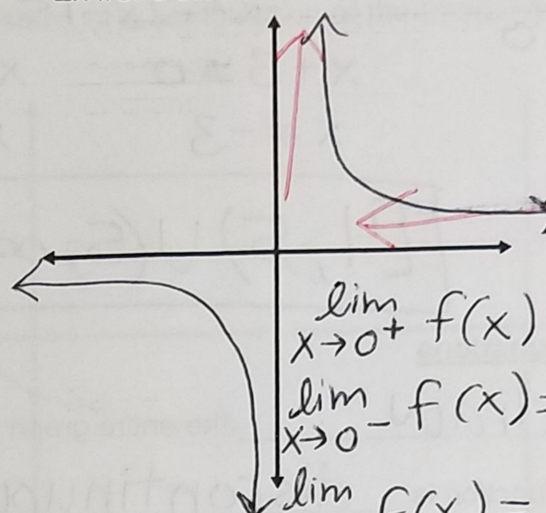
Ex #5: Determine the limits where $f(x) = x^2$



$\lim_{x \to 2} f(x) = 4$

$\lim_{x \to -3} f(x) = 9$

Ex #6: Determine the limits where $f(x) = \frac{1}{x}$



$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 0^-} f(x) = -\infty$

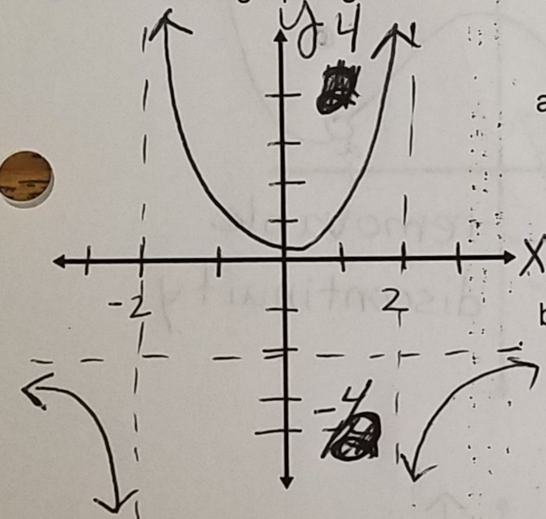
$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to 0} f(x)$ DNE

Asymptotes

Ex #7: Use a graphing calculator to graph $f(x) = \frac{2x^2}{4-x^2}$. Sketch below.

$\hookrightarrow$ 2x^2 ÷ (4−x^2)



a) Determine the horizontal asymptote and write them as limits.

$y = -2$

$\lim_{x \to \infty} = -2$

$\lim_{x \to -\infty} = -2$

b) Determine the vertical asymptote(s) and write them as limits.

$x = 2$ $\quad \lim_{x \to 2^-} f(x) = \infty \quad \lim_{x \to 2^+} f(x) = -\infty$

$x = -2$ $\quad \lim_{x \to -2^-} f(x) = -\infty \quad \lim_{x \to -2^+} f(x) = \infty$

Finding Domain Algebraically

Square Roots:

make sure stuff inside ≥ 0.

Rationals (fractions):

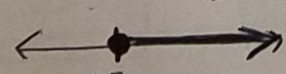
make sure is denominator $\neq 0$.

Ex #8 Find the domain of $f(x) = \sqrt{x+2}$

$x + 2 \geq 0$

$-2 \quad -2$

$x \geq -2$ inequality notation



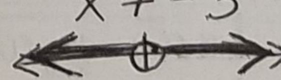
$\boxed{[-2, \infty)}$ interval notation

Ex #9: Find the domain of $f(x) = \frac{x+2}{x+3}$

$x + 3 \neq 0$

$-3 \quad -3$

$x \neq -3$



$\boxed{(-\infty, -3) \cup (-3, \infty)}$

Ex #4: Find the domain of $f(x) = \frac{-\sqrt{x-4}}{x^2-2x-15} = \frac{-\sqrt{x-4}}{(x+3)(x-5)}$

$x-4 \geq 0$

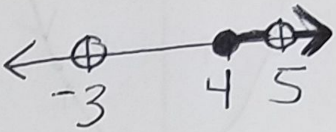
$x \geq 4$

$x+3 \neq 0$

$x \neq -3$

$x-5 \neq 0$

$x \neq 5$



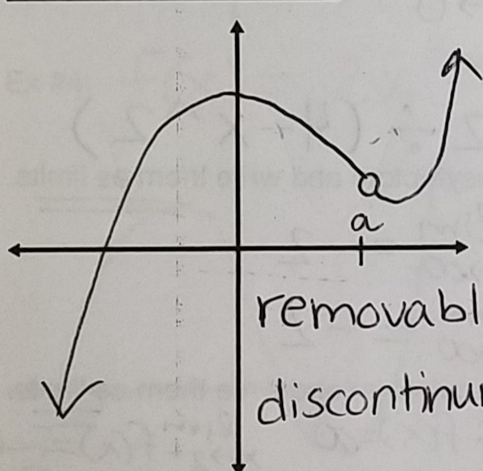
$\boxed{[4,5) \cup (5,\infty)}$

Continuous Functions

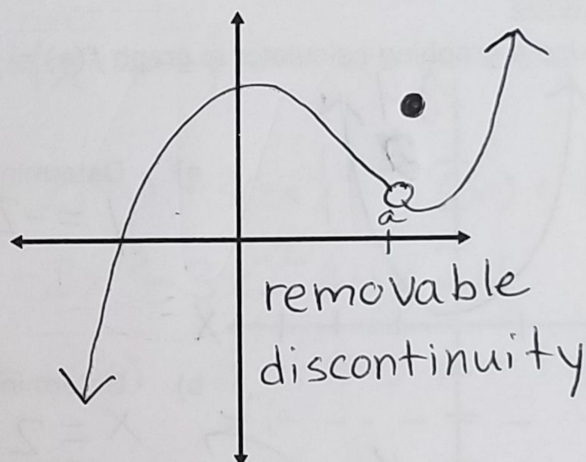
You can <u>draw</u> the entire graph <u>without taking pencil off</u>

otherwise the function is <u>discontinuous</u> at the location where the pencil came off of the paper.

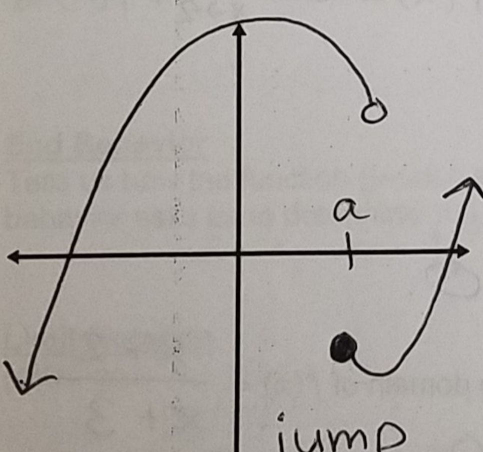
Types of Discontinuity



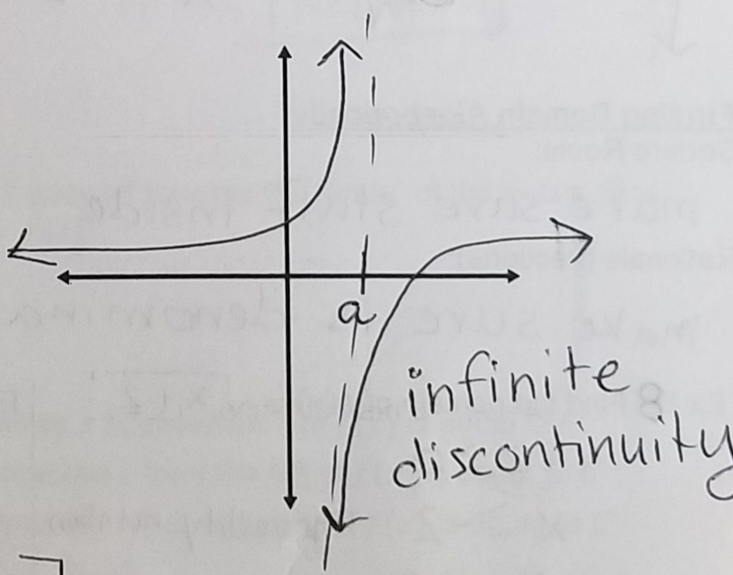
removable discontinuity



removable discontinuity



jump discontinuity

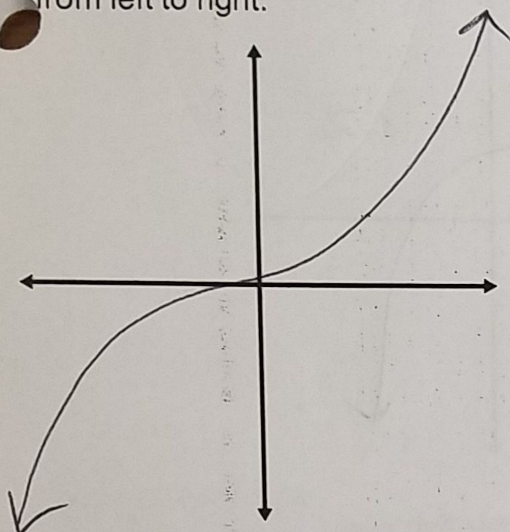


infinite discontinuity

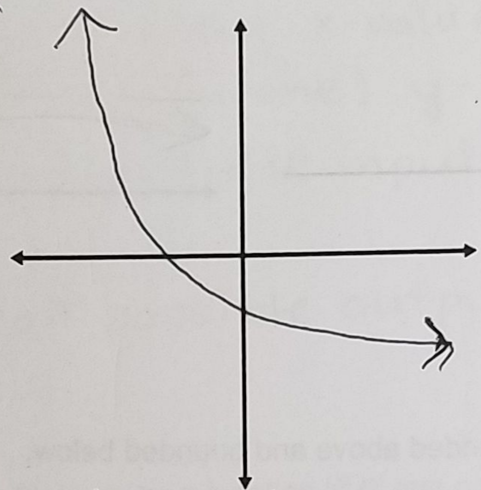
A function is continuous at $x = a$ iff $f(a) = \lim_{x \to a} f(x)$.

Increasing and Decreasing Functions

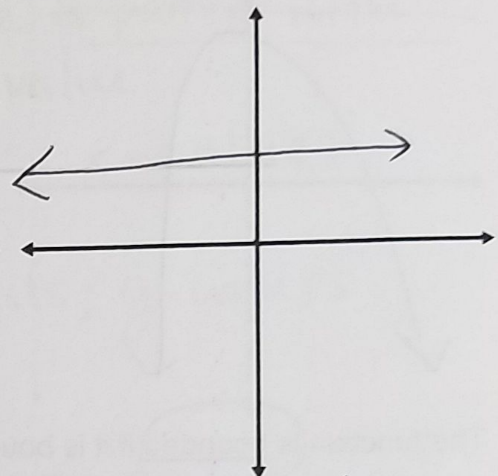
Functions are either decreasing, increasing, constant, or a combination of the three. Read the graph from left to right.



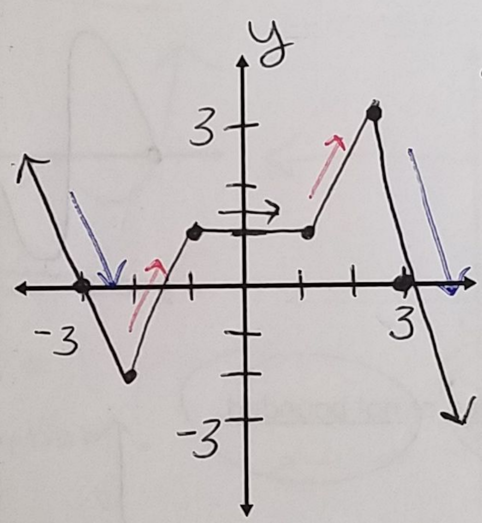
increasing



decreasing



constant



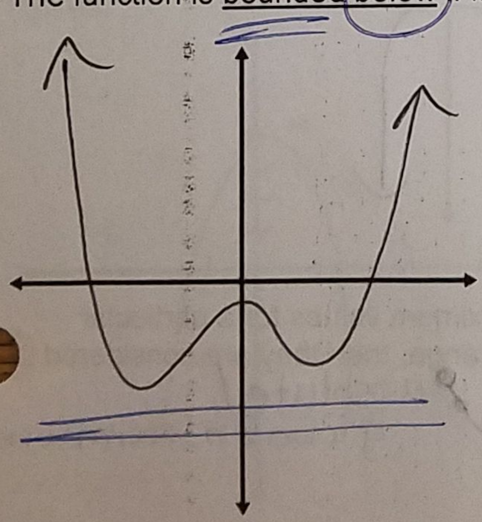
increasing:
$(-2,-1) \cup (1,2)$

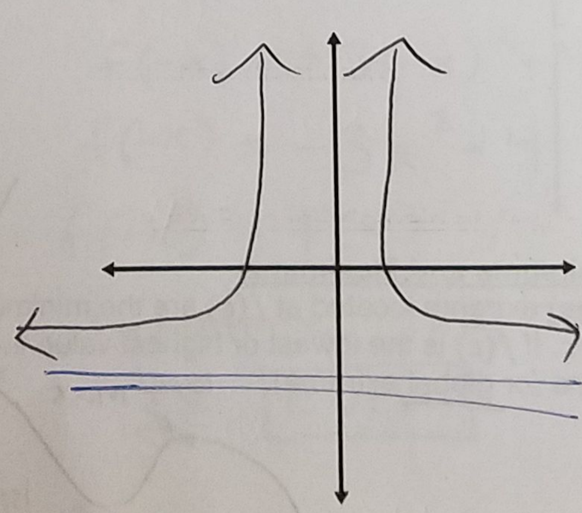
decreasing:
$(-\infty,-2) \cup (2,\infty)$

constant:
$(-1,1)$

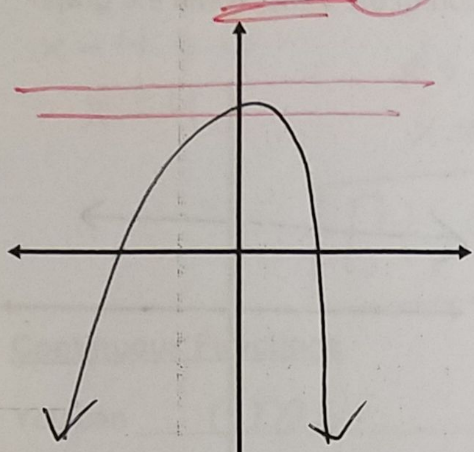
Boundedness

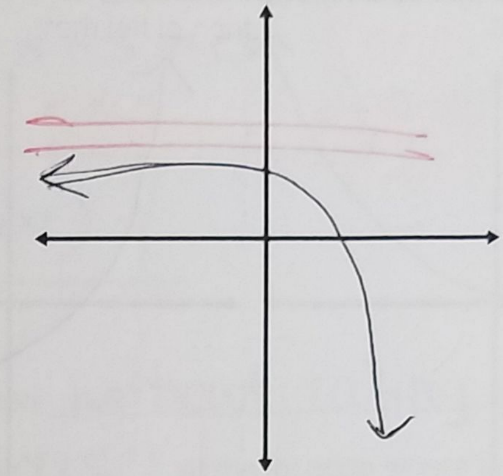
The function is bounded below if it doesn't go lower than a certain number.



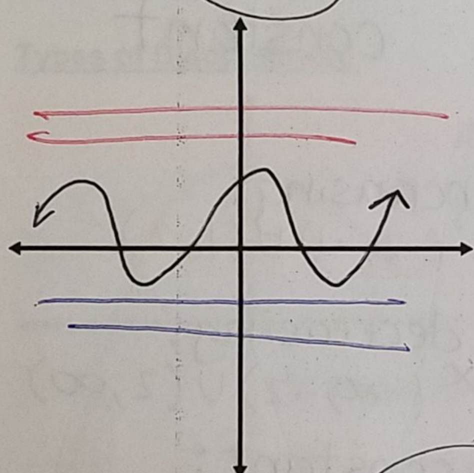


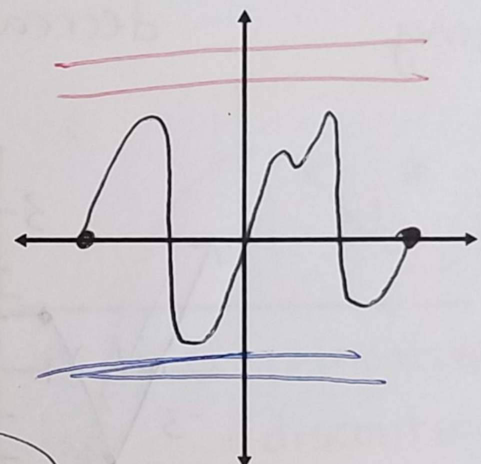
The function is bounded above if it doesn't go higher than a certain number.



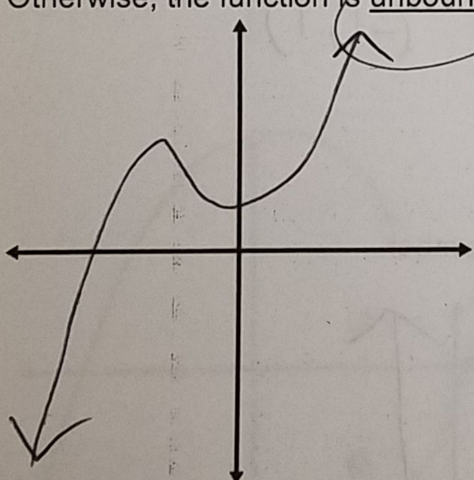


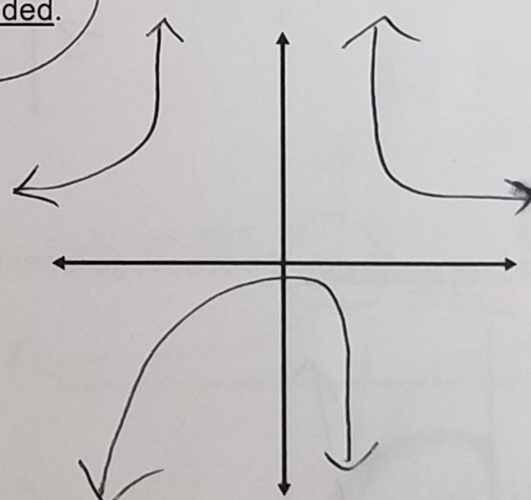
The function is bounded if it is bounded above and bounded below.





Otherwise, the function is unbounded or not bounded.





Extrema – Minimums and Maximums

Local (or relative) extrema located at $f(c)$ are the minimum and maximum values for a particular interval around c. If $f(c)$ is the lowest or highest value in the entire range, then they are considered absolute extrema (or global extrema).