

21.1 Exponential Properties & Patterns

Exponential Properties

★ $b \neq 0$ ★

NAME OF PROPERTY	ALGEBRA	EXAMPLE
ZERO EXPONENT RULE	$b^0 = 1$	$(57,410.4)^0 = 1$
NEGATIVE EXPONENT RULE	$b^{-n} = \frac{1}{b^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ OR $\frac{y}{x^{-4}} = yx^4$
PRODUCT RULE	$b^n \cdot b^m = b^{n+m}$	$2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$
QUOTIENT RULE	$\frac{b^n}{b^m} = b^{n-m}$	$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$ $\frac{2^3}{2^2} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2} = 2$
POWER RULE	$(b^n)^m = b^{n \cdot m}$	$(2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$ $(2^2 \times 2^2 \times 2^2) = (2 \cdot 2 \times 2 \cdot 2 \times 2 \cdot 2) = 2^6$
POWER OF PRODUCT RULE	$(a \cdot b)^m = a^m \cdot b^m$	$(3x)^2 = 3^2 x^2 = 9x^2$ $(3 \times 3 \times x \times x) = 9x^2$
POWER OF QUOTIENT RULE	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{4x}{3y}\right)^2 = \frac{4^2 x^2}{3^2 y^2} = \frac{16x^2}{9y^2}$ $\left(\frac{4x}{3y} \times \frac{4x}{3y}\right) = \frac{16x^2}{9y^2}$
RATIONAL (FRACTIONAL) EXPONENT RULE	$b^{\frac{m}{n}} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$	$16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$ $\sqrt[2]{16^3} = \sqrt[2]{4096} = 64$ \uparrow better \uparrow harder

- ① Zero Exponents
 - ② Power Rule
 - ③ Product Rule
 - ④ Quotient Rule
 - ⑤ Negative Exponents
- } Suggested Order

Ex #1 $(4x)^0 (y)^4 \cdot (2)(y)^2 = 4(1)(y)^4 (2)(y)^2 = 8y^6$

Ex #2 $(xy^2)^0 = 1$

Ex #3 $\frac{-3xy}{-2(y)^0} = \frac{+3xy}{+2} = \frac{3xy}{2}$

$\checkmark 6 = 4 + 2$

$$\begin{aligned}
 \text{Ex\#4 } (a^3 b^5 (-ab^5)^{-4})^4 &= a^{12} b^{20} (-ab^5)^{-16} \\
 &= a^{12} b^{20} (-1)^{-16} (a)^{-16} (b)^{-80} \\
 &= a^{-4} b^{-60} (-1)^{-16} \\
 &= \frac{1}{a^4 b^{60} (-1)^{16}} = \boxed{\frac{1}{a^4 b^{60}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex\#5 } \left(\frac{(x^{-1} y^{-1})^3}{2y^{-2} \cdot x^2 y^3} \right)^2 &= \frac{(x^{-1} y^{-1})^6}{2^2 y^{-4} x^4 y^6} = \frac{x^{-6} y^{-6}}{4 y^{-4} x^4 y^6} \\
 &= \frac{x^{-6} y^{-6}}{4 x^4 y^2} = \frac{x^{-10} y^{-8}}{4} = \boxed{\frac{1}{4x^{10} y^8}}
 \end{aligned}$$

	RATE OF CHANGE = slope	PATTERN FOR THE Y-VALUES	WHAT THE TABLE CAN LOOK LIKE	WHAT GRAPHS CAN LOOK LIKE												
LINEAR	Constant	Add same number AKA arithmetic	<table border="1"> <thead> <tr><th>X</th><th>Y</th></tr> </thead> <tbody> <tr><td>1</td><td>4</td></tr> <tr><td>2</td><td>12</td></tr> <tr><td>3</td><td>20</td></tr> <tr><td>4</td><td>28</td></tr> <tr><td>5</td><td>36</td></tr> </tbody> </table>	X	Y	1	4	2	12	3	20	4	28	5	36	
X	Y															
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2	12															
3	20															
4	28															
5	36															
EXPONENTIAL	Increasing or decreasing	Multiply same number AKA geometric	<table border="1"> <thead> <tr><th>X</th><th>Y</th></tr> </thead> <tbody> <tr><td>1</td><td>4</td></tr> <tr><td>2</td><td>8</td></tr> <tr><td>3</td><td>16</td></tr> <tr><td>4</td><td>32</td></tr> <tr><td>5</td><td>64</td></tr> </tbody> </table>	X	Y	1	4	2	8	3	16	4	32	5	64	
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Ex#6 Complete the table so that it is linear.

X	5	10	15	20	25
Y	1.25	1.00	0.75	0.50	0.25

Ex#7 Complete the table so that it is exponential.

X	0	1	2	3	4
Y	24	12	6	3	1.5