3.2 Exponential Modeling

Exponential Model

growth/decay rate

$$\underbrace{P(t)}_{\text{final value}} = \underbrace{P_0}_{\text{initial value}} (1 \underbrace{\pm r}_{\text{rate as a decimal}})^{t} \leftarrow \text{time}$$

If r is positive = growth
If r is negative = decay

"r" = growth/decay factor

Radioactive Decay

Ex #1 There are 5 grams of a substance and its half-life is 20 days.

a) Write an equation that represents the amount of substance left after t days.

★ must write the exponent in terms of the # of half-lives elapsed ★

0 halflives = 0 days = all of substance = 5g.
1 halflife = 20 days = 50% of substance = 2.5g
2 halflives = 40 days = 25% of substance = 1.25g
3 halflives = 60 days = 12.5% = $\frac{1}{8}$ = .625g

$P(t) = 5(1-.5)^{t/20}$

$\boxed{P(t) = 5(.5)^{t/20} \text{ grams}}$

b) At what time will there be 1 gram left?

$1 = 5(.5)^{t/20}$

$y_1 = 1$ $\quad y_2 = 5(.5)^{t/20}$

intersection @ (46.438562, 1)

$\boxed{\text{On the } 47^{th} \text{ day, there will be 1 gram left.}}$

Ex #2 A rumor spreads so that $s(t) = \frac{1200}{1+39e^{-0.9t}}$ models how many students know the rumor at the end of day t.

a) How many students know at day 0?

$$s(0) = \frac{1200}{1+39e^{0}} = \frac{1200}{40} = \boxed{30 \text{ students}}$$

b) When will 1000 students know?

$$1000 = \frac{1200}{1+39e^{-0.9t}}$$

$y_1 = 1000$ $\quad y_2 =$ 1200/(1+ (39(e^(-0.9t))))

$t \approx 5.859$

$\boxed{\text{On the } 6^{th} \text{ day 1000 students know.}}$