

3.1 Exponential & Logarithmic Functions

Exponential Functions

$$f(x) = a \cdot b^x$$

$a \rightarrow$ nonzero, constant, initial value, $f(0) = a$

$b \rightarrow$ positive, $\neq 1$, base

x has to be in the exponent

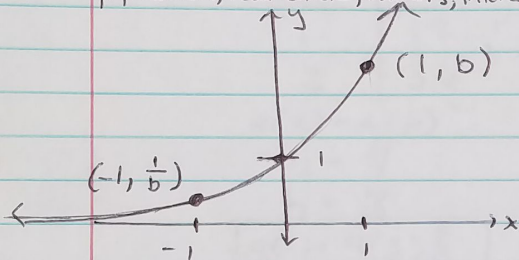
Growth & Decay

Growth

$$a > 0$$

$$b > 1$$

population, real estate, rumors, interest



$$D: (-\infty, \infty)$$

$$R: (0, \infty)$$

$$HA: y = 0$$

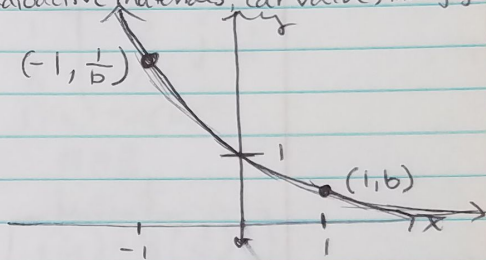
$$\lim_{x \rightarrow -\infty} f(x) = 0 \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

Decay

$$a > 0$$

$$0 < b < 1$$

radioactive materials, car value, mortgage



$$D: (-\infty, \infty)$$

$$R: (0, \infty)$$

$$HA: y = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow \infty} f(x) = 0$$

Transformations

$$f(x) = \pm a (b)^{x-c} + d$$

vertical st. compress \nearrow $x-c$ \nwarrow left/right
 \uparrow reflection over x -axis \uparrow base \uparrow up/down

Euler's Number

$$e \approx 2.71828\dots$$

Not a variable! Like π (Part of Pe^{rt})

Ex #1 Describe the transformations of $f(x) = 4(2)^{5-x} + 7 = 4(2)^{-x+5} + 7$

- vertical stretch by a factor of 4
- shift 5 units to right.
- reflection over y-axis.
- shift 7 units up.

$$\begin{aligned} -x+5 &= 0 \\ x &= 5 \\ x &\rightarrow 5 \end{aligned}$$

Ex #2 Determine the formulas from the tables.

x	g(x)
-2	4/9
-1	4/3
0	4
1	12
2	36

$$g(x) = a(b)^x$$

$$a = 4$$

$$b = 3$$

$$g(x) = 4(3)^x$$

x	f(x)
-2	128
-1	32
0	8
1	2
2	1/2

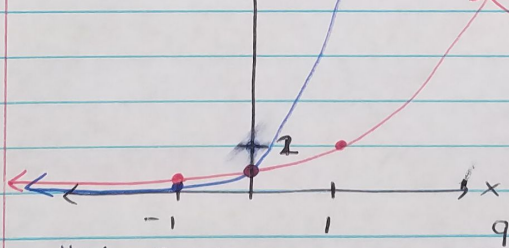
$$f(x) = a(b)^x$$

$$a = 8$$

$$b = \frac{1}{4}$$

$$f(x) = 8\left(\frac{1}{4}\right)^x$$

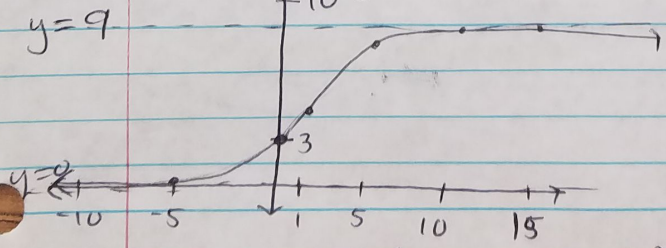
Ex #3 When is $12^x < 2^x$?



$$(-\infty, 0)$$

blue less than red

Ex #4 Analyze $y = \frac{1}{1+2e^{-x}}$.



$$D: (-\infty, \infty) \quad R: (0, 1/2)$$

$$HA: y = 1/2 \quad VA: \text{None}$$

Increasing: $(-\infty, \infty)$

Decreasing: NEVER

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = 1/2$$

Symmetry: neither

Boundedness: Bounded

continuity: cont on domain

extrema: None