3.5 Equation Solving and Modeling

One-to-One Properties

If $b^u = b^v$, then $u = v$.

If $\log_b u = \log_b v$, then $u = v$.

Solving Exponential Equations

$$\frac{a(b)^x}{a} = \frac{m}{a}$$

If you cannot use a One-to-One Property:

① Isolate the exponential, getting rid of coeffs.

② Log both sides with the <u>same</u> base as the exponent.

③ $\log_b b^u = u$, just solve normally from there.

Solving Logarithmic Equations

If you cannot use a One-to-One Property:

① Isolate the log so that it is only the log, its base & the input. $\log_b(x^2) \neq$ ~~~

② Rewrite the log as an exponent.

③ Solve normally.

Ex #1 Solve $3^{x^2-3x} = 81$.

$$3^{x^2-3x} = 3^4$$
$$x^2 - 3x = 4$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$\boxed{x = 4 \text{ or } x = -1}$$

Ex #2 Solve $4^{x+1} = \frac{1}{64}$.

$$4^{x+1} = 4^{-3}$$

$$x+1 = -3$$

$$\boxed{x = -4}$$

Ex #3 Solve $2^x = 30$.

$$\log_2 2^x = \log_2 30$$

$$x = \log_2 30 \quad \leftarrow \text{Exact answer}$$

Change of Base $\rightarrow$ $x = \frac{\log 30}{\log 2} = \frac{\ln 30}{\ln 2} = \frac{\log_{100,000,000} 30}{\log_{1,000,000} 2}$
(for calculator)

$$x \approx 4.907 \quad \leftarrow \text{Approx. (rounded) answer}$$

Ex #4 Solve $2e^x + 5 = 115$.

$$2e^x = 110$$

$$e^x = 55$$

$$\ln e^x = \ln 55$$

$$x = \ln 55$$

$$x \approx 4.007$$

Ex #5 Solve $\log_3 (x^2 + 1) = \log_3 (-2x)$

$$x^2 + 1 = -2x$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)(x+1) = 0$$

$$x = -1$$

Ex#6 Solve $\log_5(x^2-30) = \log_5 x$

$$x^2 - 30 = x$$
$$x^2 - x - 30 = 0$$
$$(x-6)(x+5) = 0$$
$$x = 6 \quad \& \quad x = -5$$

> $x = -5$ is an extraneous solution
> $x = 6$ is a solution

Ex#7 Solve $\ln e^x = \ln e^3 + \ln e^5$.

$$x = 3 + 5$$
$$\boxed{x = 8}$$

Alternative: $\ln e^x = \ln e^3 + \ln e^5$
$$\ln e^x = \ln(e^3 e^5)$$
$$e^x = e^8$$
$$\boxed{x = 8}$$

Ex #8 Solve $\log_2 x + \log_2(x-2) = 3$.

$\log_b x = y$
$\hookrightarrow b^y = x$

$$\log_2(x^2 - 2x) = 3$$
$$2^3 = x^2 - 2x$$
$$0 = x^2 - 2x - 8$$
$$0 = (x-4)(x+2)$$
$$x = 4 \quad or \quad x = -2$$

> $x = -2$ is an extraneous soln.
> $x = 4$ is a solution

Ex #9 Solve $\log_2(3x) = 4.5$
$$2^{4.5} = 3x$$
$$x = \frac{2^{4.5}}{3} \quad \leftarrow \text{Exact}$$
$$x \approx 7.542 \quad \leftarrow \text{Approx.}$$

Ex #10 Solve $\log_4(7x+2) + \log_4 9 = \log_4(2x)$
$$\log_4(63x+18) = \log_4(2x)$$
$$63x + 18 = 2x$$
$$61x = -18$$
$$\boxed{x = \frac{-18}{61} \text{ is ext.}}$$
$$\boxed{\text{NO solution}}$$

Ex #11 Solve $\log_3(x^2 - 6x) = 3$.
$$3^3 = x^2 - 6x$$
$$0 = x^2 - 6x - 27$$
$$0 = (x-9)(x+3)$$
$$\boxed{x = 9 \quad \& \quad x = -3}$$
$$\boxed{\text{None are ext!}}$$